3

c)(30 marks)

The characteristic equation is \( r^2 - 5r + 6 = 0 \), which factors as \( (r - 2)(r - 3) = 0 \), so the roots are \( r = 2 \) and \( r = 3 \). Therefore by Theorem 1 the general solution to the recurrence relation is 
\[
A_n = a_1 \times 2^n + a_2 \times 3^n
\]
for some constants \( a_1 \) and \( a_2 \). We plug in the initial condition to solve. Since \( A_0 = 1 \) we have 
\[
1 = a_1 + a_2 ,
\]
and since \( A_1 = 0 \) we have 
\[
0 = 2a_1 + 3a_2 .
\]
Therefore the solution is 
\[
A_n = 3 \times 2^n - 2 \times 3^n.
\]

d)(35 marks)

The characteristic equation is 
\[
r^2 - 4r + 4 = 0 ,
\]
which factors as \( (r - 2)^2 = 0 \), so there is only one root, \( r = 2 \), Therefore the solution is 
\[
an = 6 \times 2^n - 2 \times n2^n = (6 - 2n)2^n.
\]

e)(35 marks)

The characteristic equation is 
\[
r^2 + 4r + 4 = 0,
\]
which factors as \( (r + 2)^2 = 0 \), so again there is only one root, \( r = -2 \), which occurs with multiplicity 2. Therefore by Theorem 2 the general solution to the recurrence relation is 
\[
A_n = a_1 (-2)^n + a_2 n(-2)^n
\]
for some constants \( a_1 \) and \( a_2 \). We plug in the initial conditions to solve for the \( a \)'s. Since \( A_0 = 0 \) we have 
\[
0 = a_1 ,
\]
and since \( A_1 = 1 \) we have 
\[
1 = -2a_1 - 2a_2 .
\]
These linear equations are easily solved to yield \( a_1 = 0 \) and \( a_2 = -1/2 \). Therefore the solution is 
\[
A_n = \frac{(-1/2)n(-2)^n}{n*(-2)^{(n-1)}}.
\]