22.2-7. Represent the wrestlers as nodes, and the rivalries between them as edges. Run BFS from any of the nodes and visit all the nodes and update their distances from the Source. All odd distance wrestlers can fall into one group and even distance wrestlers can fall into the other. While performing BFS if you find two adjacent nodes being allocated to the same group, return false, as the division is not possible. This is the case of odd sized loops in the group, which cannot be split into a bipartite graph.

Runtime Complexity: Same as BFS $O(n+r)$.

22.2-8.

Approach 1:

Take any node, and run BFS on the tree and record the last discovered node. Then take that node and perform another BFS. The longest distance obtained is the diameter of the tree.

Runtime Complexity = $O(n)$, $n$= number of nodes

Approach 2:

Take every node and perform BFS, and maintaining a global max of distances recorded. The longest distance is diameter.

Runtime Complexity = $O(n^2)$, $n$= number of nodes

22.3-5.

For an “if and only if” relationship you must prove both sides of the equivalence relationship.

a.) Proof $\rightarrow$: If $(u,v)$ is a tree edge, then $v$ is discovered on exploring $(u,v)$. Otherwise if $(u,v)$ is a forward edge, means that $v$ is an ancestor of $u$. In either case $u.d < u.v$. Since $v$ was discovered after $u$, then it must be finished before $u$ is finished, hence $v.f < u.f$. Moreover, a vertex has to be discovered before it can be finished, so $v.d < v.f$. Therefore, $u.d < v.d < v.f < u.f$. 
Proof $\leftarrow$: If $u.d < v.d$ and $v.f < u.f$ means that $v$ was discovered after $u$, and finished before $u$. Thus edge $(u, v)$ is a tree edge if $v$ was discovered on traversing edge $(u, v)$. Otherwise $(u, v)$ is a forward edge.

b.) Proof $\rightarrow$: If edge $(u, v)$ is a back edge, then $v$ is an ancestor of $u$, so $v.d < u.d$. Since $v$ was discovered before $u$, it will finish before $v$ finished. Thus, $v.d < u.d < u.f < v.f$. If $(u, v)$ is a self loop, then $v.d = u.d$ and $u.f = v.f$.

Proof $\leftarrow$: If $u.f < v.f$ and $v.d < u.d$ then $v$ was discovered before $u$ and finished after $u$. Therefore, $v$ is an ancestor of $u$, and $(u, v)$ is a back edge.

c.) Proof $\rightarrow$: Edge $(u, v)$ is a cross edge. Therefore, there is no parental or ancestral relationship between $u$ and $v$. If $u.d < v.d$, then edge $(u, v)$ would indicate a parental relationship between $u$ and $v$ in the depth-first tree, which cannot happen by the case of cross edge. Hence, $u.d > v.d$. Similarly, we cannot have $u.d < v.f$, because this would indicate that $v$ was finished (but not discovered) after $u$ was discovered, but before $u$ was finished, which cannot happen. Therefore, $v.d < v.f < u.d < u.f$.

Proof $\leftarrow$: Since $v.f < u.d$, there is no parental or ancestral relationship between $u$ and $v$. Therefore edge $(u,v)$ is a cross edge.