Joint Decoding of Content-Replication Codes for Flash Memories

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I. INTRODUCTION

Flash memories are attractive due to their superior performances over prior mass storage technologies. However, one challenge is the data reliability as several types of noise [1] exist. One technology to combat errors is a strong error correcting code, e.g., LDPC code. Another mechanism is memory scrubbing [2], i.e., while errors accumulate in a codeword, with the next block erasure, the codeword is corrected and a new error-free codeword is written back to the memory. However, in flash memory rewriting are made in an out-of-place fashion, i.e., an updated codeword is stored at a new physical address and the original codeword remains in the memory. Those mechanisms lead to multiple copies of codewords, i.e., the content-replicated codewords problem.

In this work, we enhance the flash memory reliability by utilizing the existence of two content-replicated codewords for decoding, including an old codeword and a new codeword storing the same information. We aim at designing a joint decoder having access to both content-replicated codewords, and explore its decoding performance. This leads to reliability improvement in flash memories. We further study a new paradigm where the two content-replicated codewords have different forms for better performance.

II. PROBLEM STATEMENT

Let \( \mathcal{D} = \{0, 1, \cdots, M - 1\} \) be the message set for \( M \in \mathbb{N} \), and let \( \mathcal{X} \) and \( \mathcal{Y} \) be two alphabets of the symbols stored in a cell. Let two encoders be \( f_1 : \mathcal{D} \rightarrow \mathcal{X}^N \) and \( f_2 : \mathcal{D} \rightarrow \mathcal{X}^N \), and the desired joint decoder be \( h : \mathcal{Y}^N \times \mathcal{Y}^N \rightarrow \mathcal{D} \), where \( N \) is the length of codewords. Let \( \mathcal{P} = (\mathcal{X}, \mathcal{Y}, \mathcal{P}_{\mathcal{Y} \mid \mathcal{X}}) \) and \( \mathcal{Q} = (\mathcal{X}, \mathcal{Y}, Q_{\mathcal{Y} \mid \mathcal{X}}) \) be two independent channels.

We illustrate the model in Fig. 1. Here, \( m \) is a common message to both encoders, the \( N \)-dimensional vectors \( x_0^{N-1}(1), x_0^{N-1}(2) \in \mathcal{X}^N \) are two codewords obtained through two encoders (those encoders are not necessarily identical), and \( y_0^{N-1}(1), y_0^{N-1}(2) \) are two noisy codewords through \( \mathcal{P} \) and \( \mathcal{Q} \). The task is to design a joint decoder to reliably estimate the message \( m, \hat{m} \), giving \( y_0^{N-1}(1) \) and \( y_0^{N-1}(2) \).

The problem statement is presented below:

Definition 1. Given a \( (N, 2^{NR}) \) code, \( \mathcal{P} \) and \( \mathcal{Q} \), \( \mathcal{D} = \{0, 1, \cdots, 2^{NR} - 1\} \), \( f_1 : \mathcal{D} \rightarrow \mathcal{X}^N \) and \( f_2 : \mathcal{D} \rightarrow \mathcal{X}^N \), the task is to design a joint decoding function \( h : \mathcal{Y}^N \times \mathcal{Y}^N \rightarrow \mathcal{D} \) such that \( \text{Pr}(h(y_0^{N-1}(1), y_0^{N-1}(2)) \neq i|x_0^{N-1}(1) = f_1(i), x_0^{N-1}(2) = f_2(i)) \rightarrow 0 \) for \( i \in \mathcal{D} \) as \( N \rightarrow \infty \).

III. SOLUTIONS

For simplicity, assume \( \mathcal{P} \) and \( \mathcal{Q} \) are Binary Erasure Channels with the same parameter \( \epsilon \), and both encoders are LDPC encoders. We use the following notations for our LDPC codes: let the rate of two LDPC codes be \( \rho, \lambda \), let \( G_1, G_2 \) be the encoding matrices, and \( H_1, H_2 \) denotes their parity check matrices. Let \( y_0^{N-1}(1), y_0^{N-1}(2) \in \{0, 1, 2\}^N \) be two codewords received.

All proposed solutions follow the same outline, i.e., construct a LDPC code for the joint decoder based on the two LDPC codes used, determine its parity check matrix, and apply the conventional belief propagation decoder.

1) Joint decoder of identical content-replicated codes: The given encoding functions are identical in this case, i.e., \( G_1 = G_2 \) and \( H_1 = H_2 \).

Given \( y_0^{N-1}(1) \) and \( y_0^{N-1}(2) \), a codeword \( y_0^{N-1} \) for the joint decoder is obtained by comparing \( y_0^{N-1}(1) \) and \( y_0^{N-1}(2) \) to further eliminate erasures, i.e., for \( i = 0, 1, \cdots, N - 1 \), \( y_i = \begin{cases} \hat{y}_i, & \text{if } y_i(1) = y_i(2) = 1 \\ y_i(1) = y_i(2), & \text{otherwise.} \end{cases} \)

The parity check matrix for \( y_0^{N-1} \) is \( H_1 \). The decoding result is obtained by applying belief propagation to \( y_0^{N-1} \) with \( H_1 \) and initial erasure probability \( \epsilon^2 \).

Let \( \lambda(x) \) and \( \rho(x) \) be degree distributions for the LDPC codes used, let \( e^{BP}(\lambda, \rho) \) be its conventional threshold, and let \( e_{iden}(\lambda, \rho) \) denote the threshold for our joint decoder. The comparison of \( e_{iden}(\lambda, \rho) \) and \( e^{BP}(\lambda, \rho) \) is presented in Table I, and we have \( e_{iden} > e^{BP} \).

2) Joint decoder of content-replicated codes with identical information bits: The encoding functions are different in this case, i.e., \( G_1 \neq G_2 \) and \( H_1 \neq H_2 \), but the codewords carry identical information bits when regarding them as systematic codes, that is, two encoding functions are \( x_0^{N-1}(1) = u_0^{K-1}G_1 \) and \( x_0^{N-1}(2) = u_0^{K-1}G_2 \).

Let \( \mathcal{I}_1, \mathcal{I}_2 \subseteq \{0, 1, \cdots, N - 1\} \) be the information bit index sets for \( y_0^{N-1}(1) \) and \( y_0^{N-1}(2) \), and let \( p_1 \) and \( p_2 \) be their parity check bit index sets. Let \( y_0^{N-1}(1)_{\mathcal{I}_1} = (y_{i}(1) : i \in \mathcal{I}_1) \), i.e., information bits of \( y_0^{N-1}(1) \), and similar notations apply to \( y_0^{N-1}(2) \). Let \( g(\cdot) : \mathcal{I}_1 \rightarrow \mathcal{I}_2 \) be a one-to-one mapping. Similarly, further erasure elimination is possible for \( y_0^{N-1}(1)_{\mathcal{I}_1} \) when comparing it with \( y_0^{N-1}(2)_{\mathcal{I}_2} \), that is we define \( \hat{y}_0^{N-1}(1)_{\mathcal{I}_1} \), where \( y = \begin{cases} ? \text{if } y_i(1) = y_i(2) = 1 \\ y_i(1) \text{ otherwise.} \end{cases} \)

Then, a constructed codeword is \( y_0^{N-K-1} = \left( y_0^{N-1}(1)_{\mathcal{I}_1}, y_0^{N-1}(1)_{\mathcal{I}_1}, y_0^{N-1}(2)_{\mathcal{I}_2} \right) \). That is, \( y_0^{N-K-1} \) is constructed by extracting information bits from \( y_0^{N-1}(1) \) and \( y_0^{N-1}(2) \), and appending parity check bits from \( y_0^{N-1}(1) \) and \( y_0^{N-1}(2) \). An example is illustrated in Fig. 2.
Let $\epsilon_{BP}(\lambda, \rho)$ be the joint decoder threshold when the two LDPC codes have the same distribution functions $(\lambda, \rho)$. We can prove that $\epsilon_{BP}(\lambda, \rho) = \sup \{ \epsilon \in [0, 1] : x_\infty(x) = 0 \}$, where $x_\infty(x) = f(x, x_i(x), y_i(x))$, $y_i+1(x) = y_i+1(x)$, $x_0(x) = x_0$ and $y_0(x) = x$ for defined functions $f_1$ and $f_2$. (Due to space limitation, we omit its proof here.) We present several $\epsilon_{BP}$, $\epsilon_{iden}$, $\epsilon_{dij}$ in Table I, from which we see that $\epsilon_{BP} > \epsilon_{iden} > \epsilon_{dij}$.

**TABLE I**

<table>
<thead>
<tr>
<th>$(G_1, G_2)$</th>
<th>$\epsilon_{BP}$</th>
<th>$\epsilon_{iden}$</th>
<th>$\epsilon_{dij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3, 4)</td>
<td>0.644</td>
<td>0.8046</td>
<td>0.7549</td>
</tr>
<tr>
<td>(3, 5)</td>
<td>0.5176</td>
<td>0.7194</td>
<td>0.6807</td>
</tr>
<tr>
<td>(3, 6)</td>
<td>0.2994</td>
<td>0.6553</td>
<td>0.6270</td>
</tr>
<tr>
<td>(4, 6)</td>
<td>0.5051</td>
<td>0.7114</td>
<td>0.6285</td>
</tr>
<tr>
<td>(4, 8)</td>
<td>0.3834</td>
<td>0.6192</td>
<td>0.5581</td>
</tr>
</tbody>
</table>

3) Joint decoder of content-replicated codes with transformed information bits: The two encoding functions are related in this case. More specially, let $G_3$ be an intermediate LDPC generator matrix with the rate $\frac{1}{2}$. Similarly, let $P_i$ denote the information bit index set and parity check bit index set for codes with $G_i$, $i = 1, 2, 3$. The encoding algorithm is below, where $(x_0^{N-1})_{P_2}$ denotes the subvector $(x_i : i \in P_2)$.

1) $f_1 : x_0^{N-1}(1) = y_0^{K-1}G_1$.
2) $v_0^{K-1} = (u_0^{K-1}G_3)_{P_2}$.

REFERENCES
