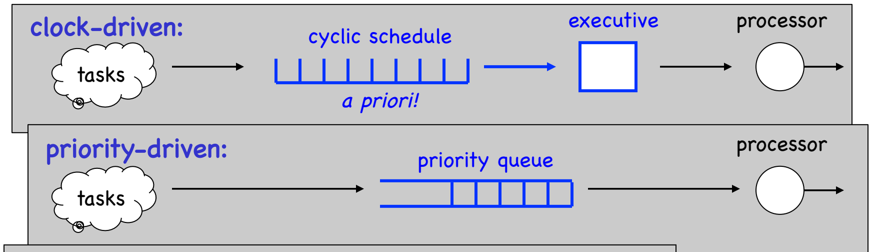
Priority-Driven Scheduling of Periodic Tasks

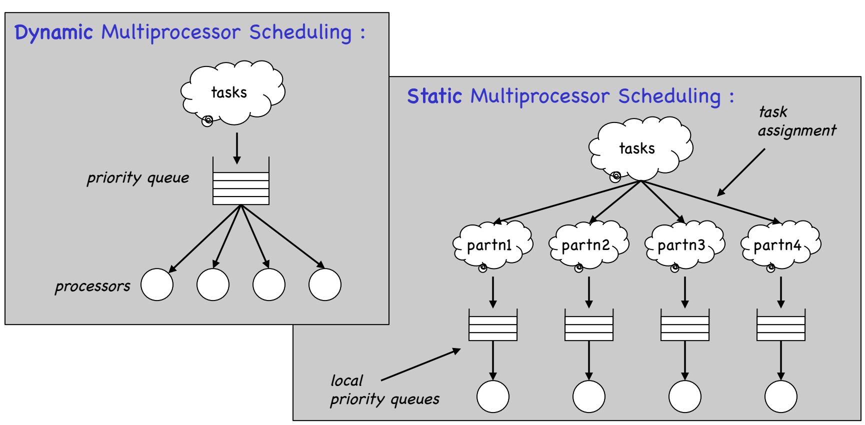
- **Priority-driven** *vs.* **clock-driven** scheduling:



- Assumptions for now:
  - tasks are periodic
  - jobs are ready as soon as they are released
  - preemption is allowed
  - tasks are independent
  - no aperiodic or sporadic tasks

- We will later:
  - integrate **aperiodic** and **sporadic** tasks
  - integrate **resources**
  - *etc.*

Why Focus on Uniprocessor Scheduling?



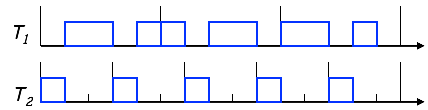
- Poor worst-case performance of priority-driven algorithms in dynamic environments.
- Difficulty in validating timing constraints.

Static-Priority *vs.* Dynamic Priority

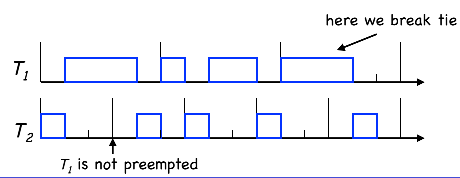
- Static-Priority: All jobs in task have same priority.
- Example: Rate-Monotonic:
  "The shorter the period, the higher the priority."

$T_1 = (5, 3, 5)$

$T_2 = (3, 1, 3)$



- Dynamic-Priority: May assign different priorities to individual jobs.
- Example: Earliest-Deadline-First:
  "The nearer the absolute deadline, the higher the priority."



Example Algorithms

- Static-Priority:
  - Rate-Monotonic (RM): "The shorter the period, the higher the priority." [Liu+Layland '73]
  - Deadline-Monotonic (DM): "The shorter the relative deadline, the higher the priority." [Leung+Whitehead '82]

- For arbitrary relative deadlines, DM outperforms RM.

- Dynamic-Priority:
  - EDF: Earliest-Deadline-First.
  - LST: Least-Slack-Time-First.
  - FIFO/LIFO
  - *etc.*

Considerations about Priority Scheduling

Question: What makes for a good scheduling algorithm?

> Def: [Schedulable Utilization]
> Every set of periodic tasks with total utilization less or equal than the schedulable utilization of an algorithm can be feasibly scheduled by that algorithm.

Two observations:

1. The **higher** the schedulable utilization, the **better** the algorithm.
2. Schedulable utilization is always less or equal 1.0!

Schedulable Utilization of FIFO

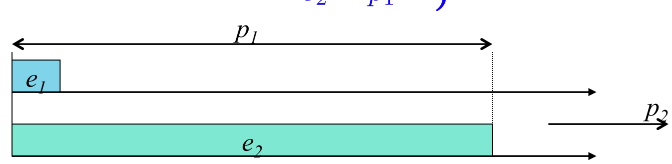
> Theorem: $U_{FIFO} = 0$

Proof:

Given **any** utilization level $\varepsilon > 0$, we can find a task set, with utilization ε, that may not be feasibly scheduled according to FIFO.

Example task set:

$$\left.\begin{array}{lll} T_1 & : & e_1 = \frac{\epsilon}{2} p_1 \\ T_2 & : & p_2 = \frac{2}{\epsilon} p_1 \\ & & e_2 = p_1 \end{array}\right\} \Rightarrow U = \epsilon$$



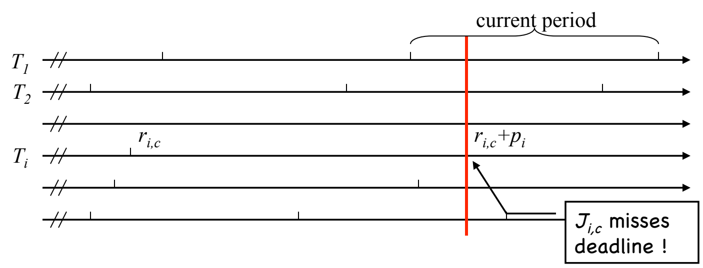
Optimality of EDF for Periodic Systems

Theorem: EDF can feasibly schedule a system of independent preemptable tasks with relative deadlines equal to their periods *iff* their total utilization is less or equal 1 .

- Proof: *only-if* : obvious
  if : show that if EDF fails to find feasible schedule, then the total utilization must exceed 1.
- Assumptions:
  - At some time t, Job $J_{i,c}$ of Task T_i misses its deadline.
  - *WLOG*: if more than one job have deadline t, break tie for $J_{i,c}$.

Optimality of EDF (cont)

- Case 1: Current period of every task begins at or after $r_{i,c}$.
- Case 2: Current period of some task may start before $r_{i,c}$.
- Case 1:

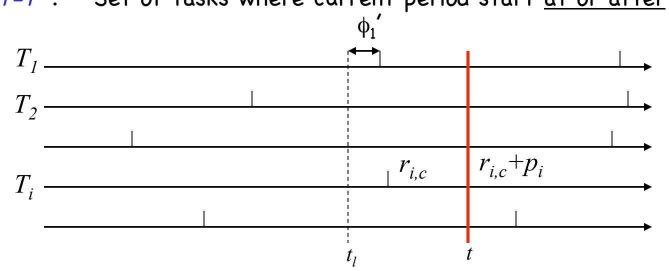


- Current jobs other than $J_{i,c}$ do not execute before time t.

$$\begin{aligned} t &< \frac{(t-\phi_i)e_1}{p_i} + \sum_{k\neq i} \left\lfloor \frac{t-\phi_k}{p_k} \right\rfloor e_k \\ &\leq t \cdot \frac{e_i}{p_i} + t \cdot \sum_{k \neq i} \frac{e_k}{p_k} \\ &= t \cdot U \\ \Rightarrow \quad & U > 1 \end{aligned}$$

Optimality of EDF (cont 2)

- Case 2:
  Some current periods start before $r_{i,c}$.
- Notation:
  - T : Set of all tasks.
  - T' : Set of tasks where current period starts before $r_{i,c}$.
  - $T-T'$: Set of tasks where current period start at or after $r_{i,c}$.

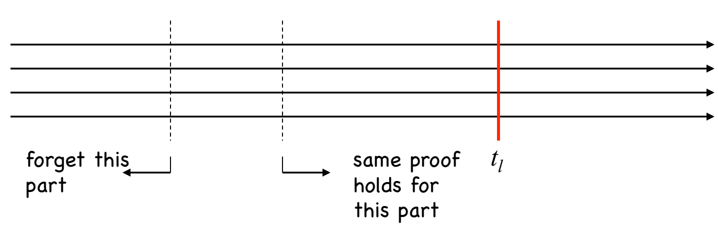


- t_l : Last point in time before t when some current job in T' is executed.
- No current job is executed immediately after time t_l.
- Why? 1. All jobs in T' are done.
  2. Jobs in $T-T'$ not yet ready.

Case 2 (cont)

$$\begin{aligned} t - t_l &< \frac{(t-t_l-\phi_i')e_1}{p_i} + \sum_{k\neq i} \left\lfloor \frac{t-t_l-\phi_k}{p_k} \right\rfloor e_k \\ &\leq (t-t_l)\cdot\frac{e_i}{p_i} + (t-t_l)\cdot\sum_{k\neq i}\frac{e_k}{p_k} \\ &= (t-t_l)\cdot U \\ \Rightarrow\quad & U > 1 \end{aligned}$$

- What about assumption that processor never idle?



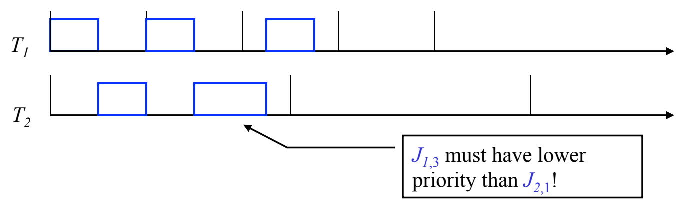
Q.E.D.

What about Static Priority?

- Static-Priority is not optimal!
- Example:

$T_1 = (2, 1, 2)$
$T_2 = (5, 2.5, 5)$

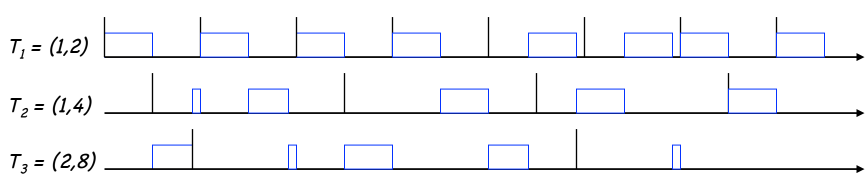
$$U = \frac{e_1}{p_1} + \frac{e_2}{p_2} = 1 \leq 100\%$$



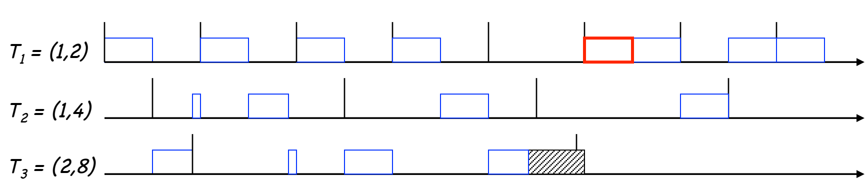
- So: Why bother with static-priority?
  - simplicity
  - predictability

Unpredictability of EDF Scheduling

- Over-running jobs in EDF hold on to their priorities
- Example

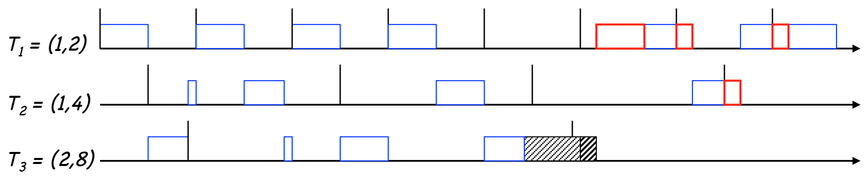


Normal Operation

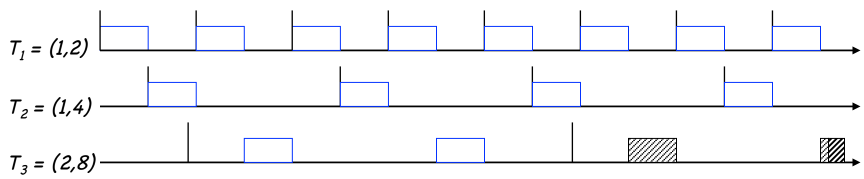


T3 over-runs by a bit more than one time unit

Unpredictability of EDF Scheduling (II)



T3 over-runs for a bit longer....



The same situation using Rate-Monotonic Scheduling:
high-priority tasks are protected

Schedulability Bounds for Static-Priority

Definition: A set of tasks is **simply periodic** if, for every pair of tasks, one period is multiple of another period.

Theorem: A system of simply periodic, independent, preemptable tasks whose relative deadlines are equal to their periods is schedulable according to RM *iff* their total utilization does not exceed 100%.

Proof: Assume T_i misses deadline at time t.
t is integer multiple of p_i.
t is also integer multiple of p_k, $\forall p_k < p_i$.

=> total time to complete jobs with deadline t :

Utilization due to i highest-priority tasks

$$\sum_{k=1}^{i} \frac{t \cdot e_k}{p_k} = t \cdot U_i = t \cdot \sum_{k=1}^{i} \frac{e_k}{p_k}$$

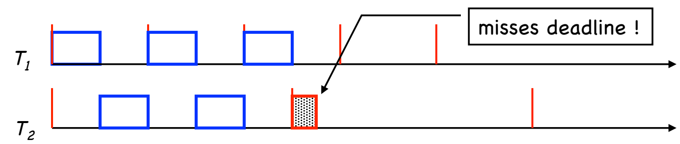
If job of T_i misses deadline, then $U_i > 1 \Rightarrow U > 1$.

Q.E.D.

Schedulable Utilization of Tasks with $D_i = p_i$ with Rate-Monotonic Algorithm

Theorem: [Liu&Layland '73] A system of n independent, preemptable periodic tasks with $D_i = p_i$ can be feasibly scheduled by the RM algorithm if its total utilization U is less or equal to $U_{RM}(n) = n(2^{1/n} - 1)$.

Why not 1.0? Counterexample: T_1 = (2, 1, 2)
T_2 = (5, 2.5, 5)



Proof Outline: First, show that theorem is correct for special case where longest period $p_n < 2p_1$ (p_1 = shortest period). We will remove this restriction later.

Proof of Liu&Layland

- General idea: Find the **most-difficult-to-schedule** system of n tasks among all **difficult-to-schedule** systems of n tasks.
- Difficult-to-schedule : Fully utilizes processor for some time interval. Any increase in execution time would make system unschedulable.
- Most-difficult-to-schedule : system with lowest utilization among difficult-to-schedule systems.
- Each of the following 4 steps brings us closer to this system.

Proof of Liu&Layland (cont)

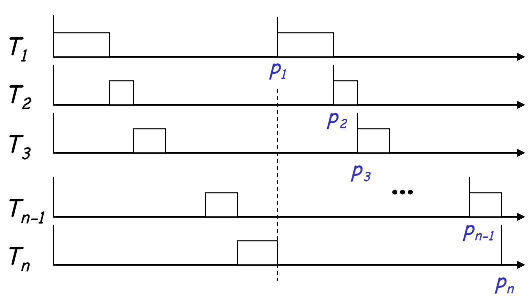
Step 1: Identify **phases** of tasks in most-difficult-to-schedule system.

System must be **in-phase**. (talk about this later)

Proof of Liu&Layland (cont)

Step 2: Choose **relationship between periods and execution times**. Hypothesize that parameters of MDTS system are thus related.

- Confine attention to first period of each task.
- Tasks keep processor busy until end of period p_n.



$$
\begin{aligned}
e_k &= p_{k+1} - p_k \\
e_n &= p_n - 2\sum_{k=1}^{n-1} e_k
\end{aligned}
$$

call this Property A

Proof Liu&Layland (cont)

Step 3: Show that any set of D-T-S tasks that are not related according to Property A has higher utilization.

- What happens if we deviate from Property A?
- Deviate one way: Increase execution of some high-priority task by ε:
  $e'_1 = e_1 + \varepsilon = p_2 - p_1 + \varepsilon$

  Must reduce execution time of some other task:
  $e'_k = e_k - \varepsilon$

$$U' - U = \frac{e'_1}{p_1} + \frac{e'_k}{p_k} - \frac{e_1}{p_1} - \frac{e_k}{p_k} = \frac{\epsilon}{p_1} - \frac{\epsilon}{p_k} > 0$$

Proof Liu&Layland (cont)

- Deviate the other way:
  Reduce execution time of some high-priority tasks by ε:

$$e''_1 = e_1 - \epsilon = p_2 - p_1 - \epsilon$$

  Must increase execution time of some lower-priority task:

$$e''_k = e_k + 2\epsilon$$

$$U'' - U = \frac{2\epsilon}{p_k} - \frac{\epsilon}{p_1} > 0$$

Proof Liu&Layland (cont)

Step 4: Express the total utilization of the M-D-T-S task system (which has Property A).

- Define $g_i := \frac{p_n - p_i}{p_i} \Rightarrow \begin{cases} e_i &= g_i p_i - g_{i+1} p_{i+1} \\ e_n &= p_n - 2 g_1 p_1 \end{cases}$

$$U = \sum_{i=1}^{n} \frac{e_i}{p_i} = \sum_{i=1}^{n-1} \left\{ g_i - g_{i+1} \frac{p_{i+1}}{p_i} \right\} + 1 - 2g_1 \frac{p_1}{p_n} = 1 + g_1 \frac{g_1 - 1}{g_1 + 1} + \sum_{i=2}^{n-1} g_i \frac{g_i - g_{i-1}}{g_i + 1}$$

- Find least upper bound on utilization: Set first derivative of U with respect to each of g_i's to zero:

$$\frac{\partial U}{\partial g_i} = \frac{g_j^2 + 2g_j - g_{j-1}}{(g_j+1)^2} - \frac{g_{j+1}}{g_{j+1}+1} = 0$$

for $j=1,2,3,\ldots,n-1$

$$g_j = 2^{(n-j)/n} - 1$$

$$\Rightarrow U = n\left(2^{1/n} - 1\right) \ .$$

Q.E.D.

What about Period Ratios > 2 ?

- We show:
  1. Every D-T-S task system *T* with period ratio > 2 can be transformed into D-T-S task system *T'* with period ratio <= 2.
  2. The total utilization of the task set decreases during the transformation step.
- We can therefore confine search to systems with period ratio < 2.
- Transformation *T->T'*: while $\exists T_k$ with $l \cdot p_k < p_n \leq (l+1)p_k \quad (l \geq 2)$

$$T_k(p_k, e_k) \rightarrow (l \cdot p_k, e_k)$$
$$T_n(p_n, e_n) \rightarrow (p_n, e_n + (l-1)e_k)$$

end

- Compare utilizations:

$$U - U' = \frac{e_k}{p_k} + \frac{e_n}{p_n} - \frac{e_k}{l \cdot p_k} - \frac{e_n + (l-1)e_k}{p_n} = \frac{e_k}{p_k} - \frac{e_k}{l \cdot p_k} - \frac{(l-1)e_k}{p_n}$$
$$= \left(\frac{1}{l \cdot p_k} - \frac{1}{p_n}\right)(l-1)e_k > 0$$

Q.E.D.

That Little Question about the Phasing...

Definition: [Critical Instant]

[Liu&Layland] If the maximum response time of all jobs in T_i is less than D_i, then the job of T_i released in the critical instant has the maximum response time.

[Baker] If the response time of some jobs in T_i exceeds D_i, then the response time of the job released during the critical instant exceeds D_i.

Theorem: In a fixed-priority system where every job completes before the next job in the same task is released, a critical instant of a task T_i occurs when one of its jobs $J_{i,c}$ is released at the same time with a job of every higher-priority task.

Proof (informal)

- Assume: Theorem holds for $k < i$.
- WLOG: $\forall k < i : \phi_k = 0$, and we look at $J_{i,1}$:
- Observation: The completion time of higher-priority jobs is independent of the release time of $J_{i,1}$.
- Therefore: The sooner $J_{i,1}$ is released, the longer it has to wait until it is completed.

Q.E.D.

Proof 2 (less informal)

- WLOG: $\min\{\phi_k \mid k = 1, \ldots, i\} = 0$
- Observation: Need only consider time processor is busy executing jobs in $T_1, T_2, \ldots, T_{i-1}$ before ϕ_i.
  If processor idle or executes lower-priority jobs, ignore that portion of schedule and redefine the ϕ_k's.
- Let $R_{i,1}$ be the response time for $J_{i,1}$.
- During $[\phi_k, \phi_i + R_{i,1}]$ a total of $\lceil (R_{i,1} + \phi_i - \phi_k) / p_k \rceil$ jobs of T_k become ready for execution.
- so (time-demand analysis): $$R_{i,1} + \phi_i = e_i + \sum_{k=1}^{i-1} \lceil \frac{R_{i,1} + \phi_i - \phi_k}{p_k} \rceil e_k$$
- and: $$R_{i,1} = e_i + \sum_{k=1}^{i-1} \lceil \frac{R_{i,1} + \phi_i - \phi_k}{p_k} \rceil e_k - \phi_i$$

Optimal Static-Priority Scheduling

Theorem: If a task set can be feasibly scheduled by some static-priority algorithm, it can be feasibly scheduled by DM.

[J.Y.-T.Leung, J. Whitehead, "On the complexity of Fixed-Priority Scheduling of Periodic, Real-Time Tasks", Performance Evaluation 2, 1982.]

Proof:

- Assume: A feasible schedule S exists for a task set T. The priority assignment is $T_1, T_2, \ldots, T_n$.
  For some k, we have $D_k > D_{k+1}$.
- We show that we can swap the priority of T_k and T_{k+1} and the resulting schedule – call it $S(k)$ – remains feasible.

Optimality of DM: Proof (II)

- So, you want to swap the priority of T_k and T_{k+1} ?!
- Observation: Response time for each task other than T_k and T_{k+1} is the same in S and $S(k)$.
- Observation: Response time of T_{k+1} in $S(k)$ must be smaller than in S, since T_{k+1} is not delayed by T_k in $S(k)$.
- Thus: Must prove that deadline of first invocation of T_k is also met in $S(k)$. (Critical Instant)
- Let x be the amount of work done in S for all tasks in $T_1,\ldots,T_{k-1}$ during interval $[0, d_{k+1}]$.
- Note: Amount of work done in S and $S(k)$ for tasks in $T_1,\ldots,T_{k-1}$ is at most x during **any** interval of length d_{k+1}.
- We must have

$$x + e_k + e_{k+1} \leq d_{k+1}$$

Optimality of DM: Proof(III)

- Observation: Number of invocations of T_{k+1} in Schedule $S(k)$ during interval $[0, \lfloor d_k/d_{k+1} \rfloor * d_{k+1}]$ is at most $\lfloor d_k/d_{k+1} \rfloor$.
- Observation: Amount of work for all tasks in $T_1,\ldots,T_{k-1}$ in the interval $[0, \lfloor d_k/d_{k+1} \rfloor * d_{k+1}]$ is at most $\lfloor d_k/d_{k+1} \rfloor * x$.
- The following condition is sufficient ot guarantee that the deadline of the first request of T_k is met in S(k):

$$\lfloor d_k/d_{k+1} \rfloor * (x+e_{k+1}) + e_k \leq \lfloor d_k/d_{k+1} \rfloor * d_{k+1}$$

- This, however, follows from inequality on previous page. (qed)

Why Utilization-Based Tests?

- If no parameter ever varies, we could use simulation.
- But:
  - Execution times may be smaller than e_i
  - Inter-release times may vary.
- Tests are still robust.
- Useful as methodology to define execution times or periods.

Time-Demand Analysis

- Compute total demand on processor time of job released at a critical instant and by higher-priority tasks as function of time from the critical instant.
- Check whether demand can be met before deadline.
- Determine whether T_i is schedulable:
  - Focus on a job in T_i, suppose release time is critical instant of T_i:
    $w_i(t)$: Processor-time demand of this job and all higher-priority jobs released in (t_0, t):

$$w_i(t) = e_i + \sum_{k=1}^{i-1} \lceil \frac{t}{p_k} \rceil e_k$$

- This job in T_i meets its deadline if, for some

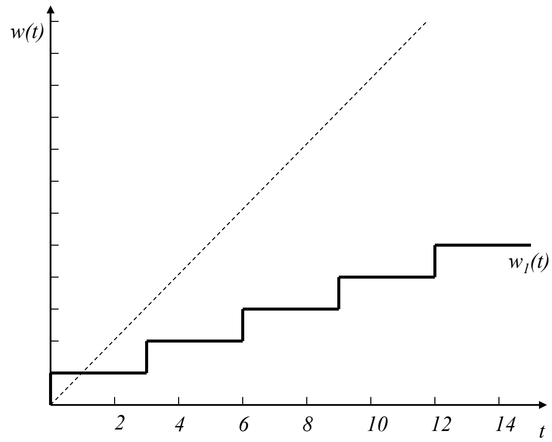
$$t_1 \leq D_i \leq p_i \quad : \quad w_i(t_1) \leq t_1$$

- If this does not hold, job cannot meet its deadline, and system of tasks is not schedulable by given static-priority algorithm.

Example

T_1	=	(3, 1)
T_2	=	(5, 1.5)
T_3	=	(7, 1.25)
T_4	=	(8, 0.5)

$$w_1(t) = \lceil \frac{t}{p_1} \rceil e_1$$

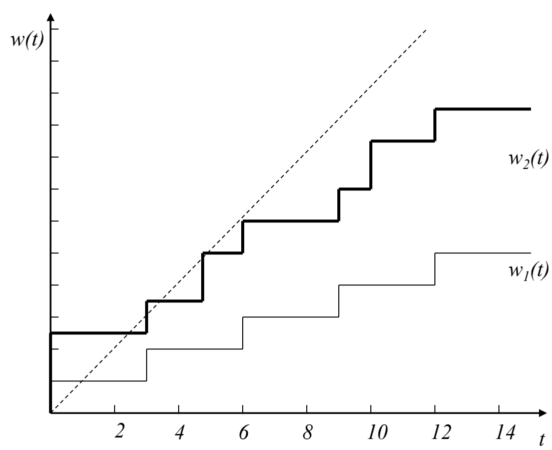


Example

T_1	=	(3, 1)
T_2	=	(5, 1.5)
T_3	=	(7, 1.25)
T_4	=	(8, 0.5)

$$w_1(t) = \lceil \frac{t}{p_1} \rceil e_1$$

$$w_2(t) = \lceil \frac{t}{p_1} \rceil e_1 + \lceil \frac{t}{p_2} \rceil e_2$$



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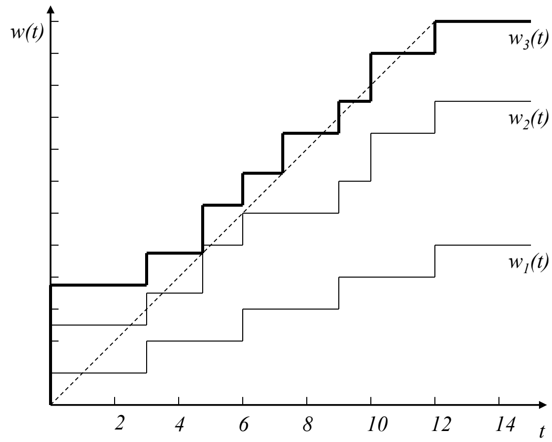

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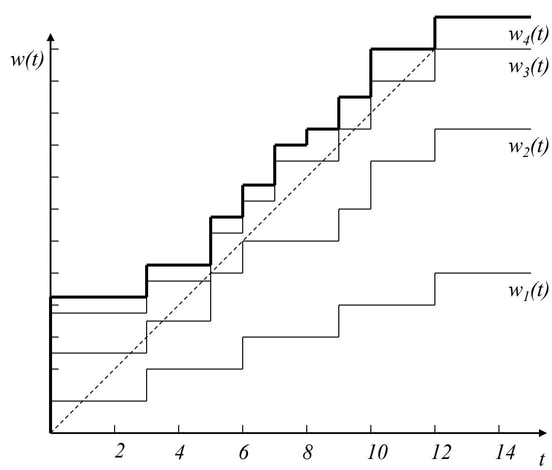


Practical Factors

- Non-Preemptable Portions (*)
- Self-Suspension of Jobs (*)
- Context Switches (*)
- Insufficient Priority Resolutions (Limited Number of Distinct Priorities)
- Time-Driven Implementation of Scheduler (Tick Scheduling)
- Varying Priorities in Fixed-Priority Systems

Practical Factors I: Non-Preemptability

- Jobs, or portions thereof, may be non-preemptable.

Definition: [non-preemptable portion] Let's denote by ρ_I the largest non-preemptable portion of jobs in T_i.

Definition: A job is said to be **blocked** if it is prevented from executing by lower-priority job. (priority-inversion)

- When testing schedulability of a task T_i, we must consider
  - higher-priority tasks

  and
  - non-preemptable portions of lower-priority tasks

Analysis with Non-Preemptable Portions

Definition: The **blocking time** b_i of Task T_i is the longest time by which any job of T_i can be blocked by lower-priority jobs:

$$b_i = \max_{i+1 \le k \le n} \rho_k$$

- Time-demand function with blocking:

$$w_i(t) = e_i + b_i + \sum_{k=1}^{i-1} \lceil \frac{t}{p_k} \rceil e_k$$

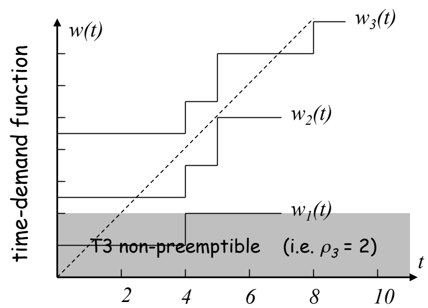
- Utilization bounds with blocking:

  test one task at a time:

$$\frac{e_1}{p_1} + \frac{e_2}{p_2} + \ldots + \frac{e_i + b_i}{p_i} = \sum_{k=1}^{i} \frac{e_k}{p_k} + \frac{b_i}{p_i} \le U_{RM}(i)$$

Non-Preemptability: Example

T_1 = (4, 1)
T_2 = (5, 1.5)
T_3 = (9, 2)



Practical Factors II: Self-Suspension

Definition: Self-suspension of a job occurs when the job waits for an external operation to complete (RPC, I/O operation).

- Assumption: We know the maximum length of external operation; i.e., the duration of self-suspension is bounded.
- Example:

$T_1 = (\phi_1=0, p_1=4, e_1=2.5)$

$T_2 = (\phi_2=3, p_2=7, e_2=2.0)$

- Analysis: b_i^{SS} : Blocking time of T_i due to self-suspension.

$$b_i^{SS} = \text{max. self-suspension time of } T_i + \sum_{k=1}^{i-1} min(e_k, \text{max. self-suspension time of } T_k)$$

Self-Suspension with Non-Preemptable Portions

- Whenever job self-suspends, it loses the processor.
- When tries to re-acquire processor, it may be blocked by tasks in non-preemptable portions.
- Analysis:
  - b^{NP}_i: Blocking time due to non-preemptable portions
  - K_i: Max. number of self-suspensions
  - b_i: Total blocking time

$$b_i = b^{SS}_i + (K_i + 1)\, b^{NP}_i$$

Practical Factors III: Context Switches

Definition: [Job-level fixed priority assignment]
In a job-level fixed priority assignment, each job is given a fixed priority for its entire execution.

- Case I: No self-suspension
  - In a job-level fixed-priority system, each job preempts at most one other job.
  - Each job therefore causes at most **two** context switches
  - Therefore: Add the context switch time twice to the execution time of job: $e_i = e_i + 2\ CS$
- Case II: Self-suspensions can occur
  - Each job suffers **two more** context switches each time it self-suspends
  - Therefore: Add more context switch times appropriately: $e_i = e_i + 2\ (K_i + 1)\ CS$