Handling Overload

(G. Buttazzo, Hard Real-Time Systems, Ch. 9)

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Causes for Overload

- Bad system design
  - e.g. poor estimation of worst-case execution times
- Simultaneous arrival of unexpected events
- Malfunctioning of input devices
  - "babbling idiot" problem
- Unpredicted variations of environmental conditions
- Operating system exceptions
  - Caused by anomalous combination of data
  - exceptions handlers may starve real-time workload

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Definitions of "Load"

In standard queueing theory:

λ = average arrival rate

μ = mean service time

$\rho = \lambda \mu$ = average load

For periodic tasks:

$$\rho = U = \Sigma C_i/T_i$$

ρ : "system load"

U : "utilization factor"

For generic set of jobs:

$$\rho(t_a, t_b) = \max_{t_1, t_2 \in [t_a, t_b]} \frac{g(t_1, t_2)}{t_2 - t_1}$$

g(t1, t2) : processor demand during interval [t1, t2]

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Instantaneous Load

Definition (Instantaneous Load):

$$\rho_i(t) = \frac{\sum_{d_k \leq d_i} c_k(t)}{(d_i - t)},$$

ρ_i = "partial load" for job J_i at time t.

$c_k(t)$ = remaining execution time of J_k a time t.

Total load at time t:

$$\rho(t) = \max_i \rho_i(t)$$

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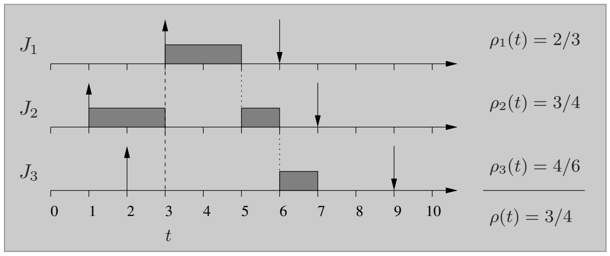
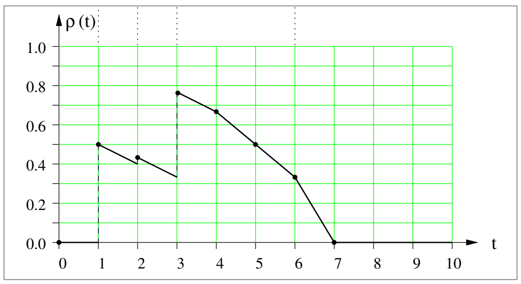
Instantaneous Load: Example



Instantaneous Load at time t = 3



Instantaneous Load Function

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Overload vs. Overrun

Definition (Overload):

A computing system experiences Overload when the computation time demanded by task set in an interval exceeds the available processing time.

Definition (Overrun):

A task (job) experiences Overrun when it exceeds its expected utilization.

Overruns may occur because of:

- Activation Overrun: job is activated before expected arrival time.
- Execution Overrun: job computation time exceeds expected value.

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Transient vs. Permanent Overload

Definition (Transient Overload):

Overload condition occurs for a limited duration in a system with average load less than schedulable utilization (*), e.g.,

$$\rho_{avg} <= 1$$
$$\rho_{max} > 1$$

Definition (Permanent Overload):

Overload condition occurs for a unpredictable duration in a system with average load higher than schedulable utilization, (*), e.g.,

$$\rho_{avg} > 1$$

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Types of Overload Conditions

Transient overloads due to aperiodic jobs:
- can happen in event-triggered systems.

Transient overloads due to task overruns:
- tasks execute (or are actived) more than expected.
- can happen in event-triggered and time-triggered systems.

Permanent overloads due in periodic task systems:
- total utilization factor is larger than schedulable utilization. (*)

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Performance Metric

- $V(f_i)$ = Value of task as function of finish time f_i
  - reflects importance of task
- Cumulative Value of Algorithm A:

$$\Gamma_A = \Sigma_i \, v(f_i)$$

- Maximum achievable cumulative value Γ^*:

$$\Gamma^* := \max_A(\Gamma_A)$$

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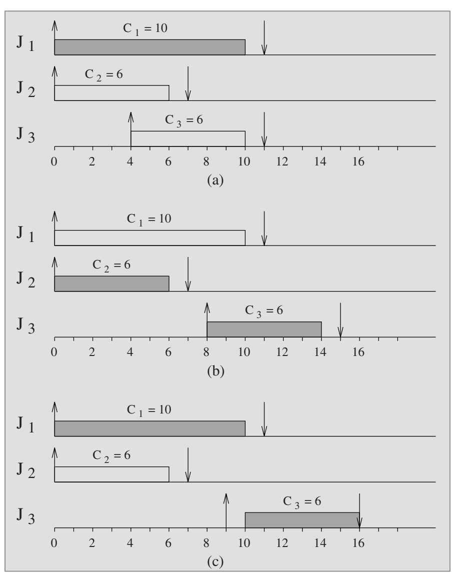
Dynamic (On-Line) vs. Clayrvoyant Schedulers

Example:



$\Gamma_{max} = 10$

$\Gamma_{max} = 12$

$\Gamma_{max} = 14$

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Competitive Factor

Question: What is the minimum cumulative value that can be achieved by an algorithm on any task set?

Definition (**Competitive Factor**):

A scheduling algorithm A has a competitive factor ϕ_A iff it can guarantee a cumulative value $\Gamma_A >= \phi_A \Gamma^*$, where Γ^* is the cumulative value achieved by the optimal clairvoyant scheduler.

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EDF has Competitive Factor Zero (ϕ_{EDF} = 0)

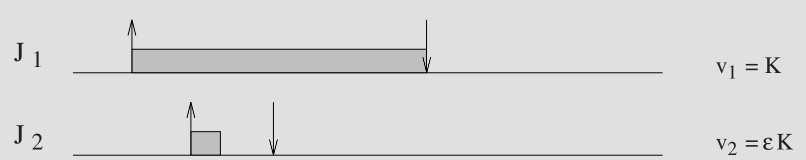

Figure 9.8 Situation in which EDF has an arbitrarily small competitive factor.

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What is the Cost of no Clairvoyance?

Theorem (Baruah *et al.*):

In a system where the loading factor is greater than 2 ($\rho > 2$) and tasks' values are proportional to their computation times ($V_i = C_i$), no on-line algorithm can guarantee a competitive factor greater than 25%.

Proof by adversarial argument:

- Scheduler is player, clairvoyant scheduler is adversary
- Adversary generates sequence of tasks to minimize Γ_A / Γ^*
- At the end of game, compare Γ_A and Γ^*

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Adversarial Argument: Task Generation

- Major tasks, of length C_i, and associated tasks, of length ε arbitrarily small.
- All tasks have zero laxity.
- After releasing a major tasks J_i, adversary releases next major task J_{i+1} at time before the deadline of J_i, that is, $r_{i+1} = d_i - \varepsilon$
- For each major task J_i, adversary may also create a sequence of associated tasks, $[r_i, d_i]$, such that each subsequent associated task is released at the deadline of the previous one in the sequence.
  - Resulting load is $\rho = 2$.
  - Any algorithm that schedules an assoc. tasks cannot schedule J_i within its deadline.
- If player schedules assoc. task, adversary stops sequence of assoc. tasks.
- If player schedules J_i, sequence of tasks stops with release of J_{i+1}
- Sequence has finite length, i.e., until J_m for some value for m.

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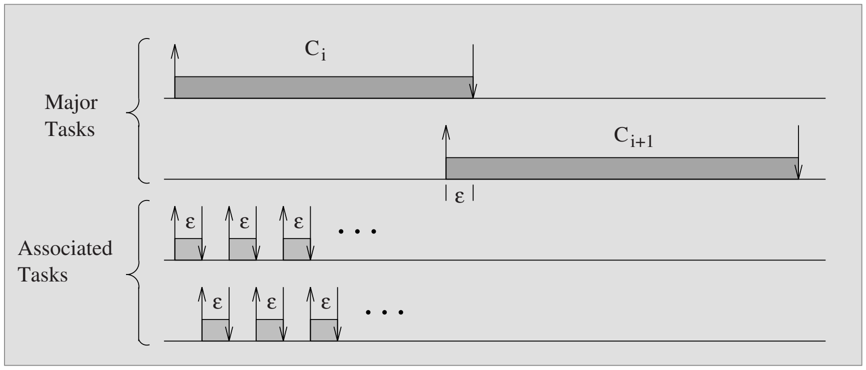
Task Generation Strategy (II)



- Player will never abandon major task for associated task. (value would be negligible)
- However, values of major tasks are chosen by adversary.
- Let $J_0, J_1, \cdots, J_i, \cdots, J_m$ be worst-case sequence of tasks, WLOG $C_0 = 1$

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Three Cases

Case 1: Player decides to schedule J_0:

- sequence terminates with J_1.
- cumulative value obtained by player is C_0
- cumulative value obtained by adversary is $C_0 + C_1 - \varepsilon$
- ratio is

$$\varphi_0 = \frac{C_0}{C_0 + C_1} = \frac{1}{1 + C_1} = \frac{1}{k}.$$

(We let $C_1 = k-1$)

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Three Cases (II)

Case 2: Player decides to schedule J_1:

- sequence terminates with J_2.
- cumulative value obtained by player is C_1
- cumulative value obtained by adversary is $C_0 + C_1 + C_2$
- ratio is

$$\varphi_1 = \frac{C_1}{C_0 + C_1 + C_2} = \frac{k-1}{k + C_2}.$$

(remember: $C_1 = k-1$)

- Observation 1: $\varphi_1 <= \varphi_0$, otherwise adversary would have "stopped earlier": $(k-1)/(k+C_2) <= 1/k$
- Observation 2: $\varphi_1 >= \varphi_0$, otherwise player would "stuck with" J_0: $(k-1)/(k+C_2) >= 1/k$
- Therefore: $\varphi_1 = \varphi_0$ => $(k-1)/(k+C_2) = 1/k$
- And: $C_2 = k^2 - 2k$

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Three Cases (III)

Case i: Player decides to schedule J_i:

- sequence terminates with J_{i+1}.
- cumulative value obtained by player is C_i
- cumulative value obtained by adversary is $C_0 + C_1 + \ldots + C_{i+1}$
- ratio is

$$\varphi_i = \frac{C_i}{\sum_{j=0}^{i} C_j + C_{i+1}}.$$

- Observation: $\varphi_i = \varphi_{i-1} = \ldots = \varphi_0 = 1/k$
- Thus,

$$\varphi_i = \frac{C_i}{\sum_{j=0}^{i} C_j + C_{i+1}} = \frac{1}{k},$$

- and

$$C_{i+1} = kC_i - \sum_{j=0}^{i} C_j.$$

Worst-case computation times:

$$\begin{cases} C_0 &= 1 \\ C_{i+1} &= kC_i - \sum_{j=0}^{i} C_j. \end{cases}$$

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So, what about the Bound?!

- $$\frac{C_{m-1}}{\sum_{j=0}^{m-1} C_i + C_m} = \frac{1}{k}$$ J_i, $$\varphi_i = \frac{C_i}{\sum_{j=0}^{i} C_j + C_{i+1}} = \frac{1}{k}.$$
- $$\Longleftrightarrow kC_{m-1} = \sum_{j=0}^{m-1} C_i + C_m$$ $$\varphi_m = \frac{C_m}{\sum_{j=0}^{m} C_j}.$$
- $$\Longleftrightarrow C_m = kC_{m-1} - \sum_{j=0}^{m-1} C_i$$
  such that $\varphi_m <= 1/k$, that is, $$\frac{C_m}{\sum_{j=0}^{m} C_j} \leq \frac{1}{k},$$
- Note:

$$\frac{C_m}{\sum_{j=0}^{m} C_j} = \frac{C_m}{\sum_{j=0}^{m-1} C_j + C_m} = \frac{C_m}{\sum_{j=0}^{m-1} C_j + kC_{m-1} - \sum_{j=0}^{m-1} C_j} = \frac{C_m}{kC_{m-1}}.$$

- Therefore: $$\frac{C_m}{\sum_{j=0}^{m} C_j} = \frac{C_{m-1}}{kC_{m-1}} \leq \frac{1}{k} \Longleftrightarrow C_m \leq C_{m-1}$$

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The Bound (II)

- Recall: $$\frac{C_m}{\sum_{j=0}^{m} C_j} = \frac{C_{m-1}}{kC_{m-1}} \leq \frac{1}{k} \Longleftrightarrow C_m \leq C_{m-1}$$
- We can rewrite: $$\begin{aligned} C_{i+2} &= kC_{i+1} - \sum_{j=0}^{i+1} C_j \\ C_{i+1} &= kC_i - \sum_{j=0}^{i} C_j, \end{aligned}$$
- $$\frac{C_{m-1}}{\sum_{j=0}^{m-1} C_i + C_m} = \frac{1}{k}$$ $$C_{i+1} = k(C_{i+1} - C_i) - C_{i+1};$$
- $$\Longleftrightarrow kC_{m-1} = \sum_{j=0}^{m-1} C_i + C_m$$ $$k(C_{i+1} - C_i).$$
- $$\Longleftrightarrow C_m = kC_{m-1} - \sum_{j=0}^{m-1} C_i$$ $$\begin{aligned} &= 1 \\ &= k - 1 \\ &= k(C_{i+1} - C_i). \end{aligned}$$

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The Bound (III)

- Recurrence Relation:

$$\begin{cases} C_0 & = & 1 \\ C_1 & = & k-1 \\ C_{i+2} & = & k(C_{i+1}-C_i). \end{cases}$$

The tightest bound on the competitive factor is given by the smallest *ratio 1/k* such that recurrence relation satisfies

$$C_m <= C_{m-1}$$

- All we need to do is solve the recurrence relation. (Standard Discrete Math)
- We get 3 cases ($k > 4$; $k = 4$; $k < 4$)
- Only Case $k < 4$ has solution.

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