## Discussion of Problem 3 in Homework 2

For the following discussion we use the notation from the non-preemptive total bandwidth server.

(a) By assumption the connection is busy between  $t_{-1}$  and t. In addition, the total length of packets arriving during this period does not exceed  $BW(t - t_{-1})$ .

The total length of packets  $(\sum e_i)$  therefore must be equal to  $BW(t - t_{-1})$ .

According to the mechanism of the Total Bandwidth Server, at the arrival of the first packet of a busy period, the virtual clock d is set as follows:

$$d := max(d_{begin}, t_{-1}) + e_1/BW .$$

We can assume that d was initially less than  $t_{-1}$ .

During the backlogged period, d is repeatedly increased by  $e_i/BW$  and reaches the maximum at the end of the backlogged period. At the end of the backlogged period, d is

$$d_{end} = max(d_b egin, t_{-1}) + \sum e_i / BW = t_{-1} + \sum e_i / BW = t_{-1} + BW(t - t_{-1}) / BW = t .$$

Accordingly,  $d_{end}$  exceeds t if  $\sum e_i$  exceeds  $BW(t - t_{-1})$ .

(b) We note that the virtual time and the current time are equal when either a busy period for a connection is starting or when a packet is just completed exactly by its "deadline" and a new packet is starting. (For the following argument, both cases are identical.)

Therefore, the argument is identical to that under (a), and the virtual time at current time t is identical to the current time t at that point.