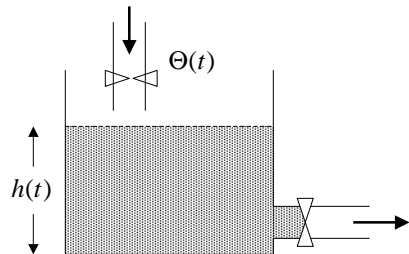


Real-Time Systems: Example / Case Studies

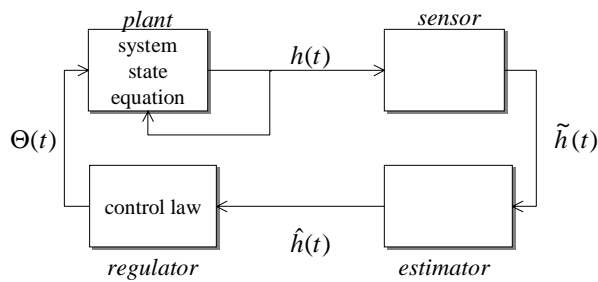
- Simple Control System
 - Sampling Periods
 - Quality of the Control vs. Processing Cost
 - Protection of Resources in Integrated Systems
 - Multimedia / Real-Time Communication
 - Synchronization of Activities:
 - Example: Stream Synchronization
 - Anomalies in Asynchronous Systems
 - Example: Advanced Fighter Technology Integration (AFTI) F16
-

Application Areas: Control Systems

- Example: Water Tank →



- In other words ↘



Control Systems (cont)

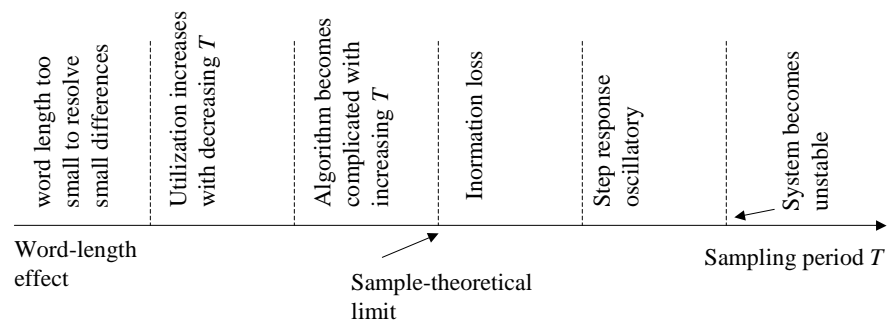
- Control Loop:

```

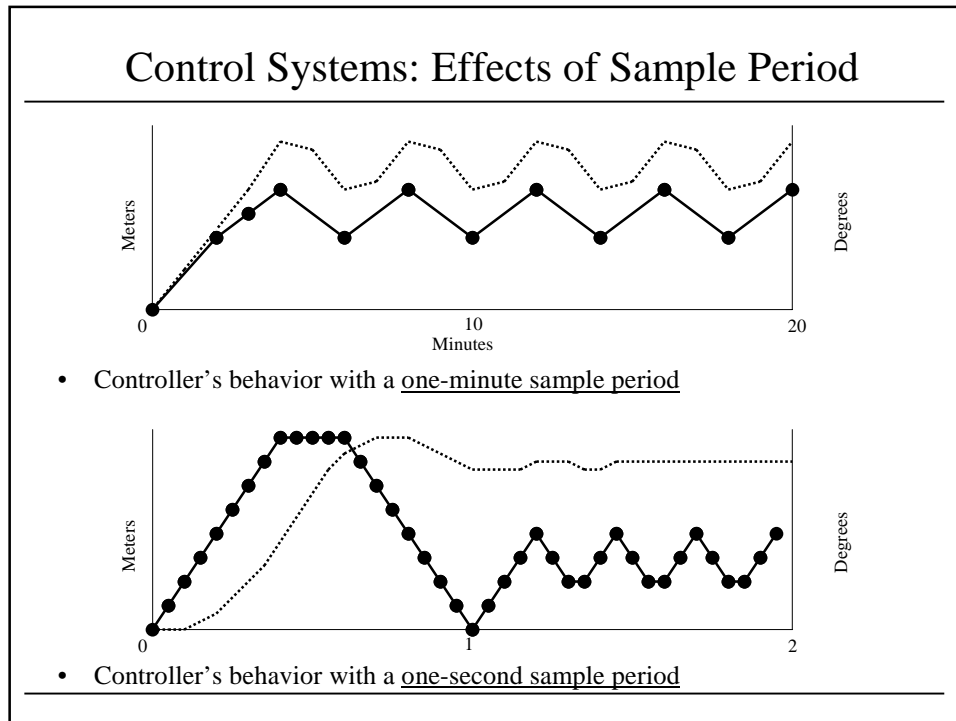
DO FOREVER
  wait_for_delay
  h      := fluid_height
  theta := valve_position
  r      := table_lookup(h, theta)
  IF     r = left THEN turn_left
  ELSE IF r = right THEN turn_right
  ELSE   do_nothing
ENDDO

```

Control Systems: Choice of Sampling Period



- Higher sampling rate is sometimes chosen to
 - reduce the delay between a command change and the plant response
 - produce smooth response



Quality of Control vs. Processing Cost

Example: Open-Loop Temperature Control

[Simplified from : Setol, Lehoczky, Sha, and Shin, "On Task Schedulability in Real-Time Control Systems", Proceeding of the 1996 IEEE Real-Time Systems Symposium]

- System: Temperature of a unit is controlled by a burner.
- Dynamic equation:

$$\dot{x} = -ax + bu$$
 - x difference between unit and ambient temperature, $x(0) = 0$
 - u control input, rate of heat
- Problem: change temperature of unit to x_d within time t_f ; consume minimum amount of fuel. Allow for a tolerance δ .

$$|x(t_f) - x_d| \leq \delta$$
- Performance Index $J(u)$ of control system: measure of total cost of control and accuracy generated in time period $[0, t_f]$ by control u . Generally:

$$J(u) = S(x(t_f), t_f) + \int_0^{t_f} L(x(t), u(t), t) dt$$
- Optimal control $u^*(t)$ with performance index J^* .

Open-Loop Temperature Control (cont)

- Our case: minimize fuel.

$$\min_u J = \frac{1}{2} p(x(t_f) - x_d)^2 + \frac{1}{2} \int_0^{t_f} u^2(t) dt$$

- Resulting optimal control:

$$u^*(t) = \frac{x_d p a b e^{at}}{a e^{at_f} + p b^2 \sinh(at_f)}$$

- Final state:

$$x^*(t_f) = \frac{x_d p b^2 \sinh(at_f)}{a e^{at_f} + p b^2 \sinh(at_f)}$$

Open-Loop Temperature Control (cont)

- Discretize control input u :
 - Sampling period P .

$$\dot{x}^*(t) = -ax^*(t) + bu^*(kP) \quad kP \leq t \leq (k+1)P$$

- Performance index for discrete optimal control:

$$J_D^*(P) = S(x^*(t_f), t_f) + \sum_{k=0}^{n-1} \int_{kP}^{(k+1)P} L(x^*(t), u^*(kP), t) dt$$

- In our case:

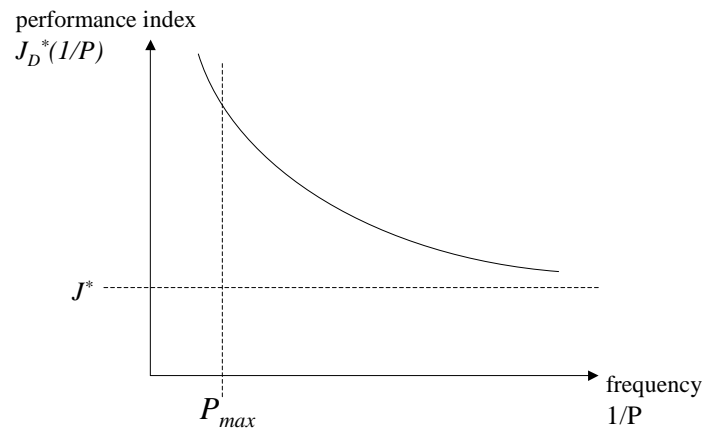
$$J_D^*(P) \cong \frac{1}{2} p x_d \left(\frac{1 - e^{-aP}}{1 + e^{-aP}} \right)^2$$

- Constraints: $|x(t_f) - x_d| \leq \delta$

$$x_d \left(\frac{1 - e^{-aP}}{1 + e^{-aP}} \right) \leq \delta \quad \Rightarrow \quad P \leq \frac{1}{a} \ln \frac{x_d + \delta}{x_d - \delta}$$

Open-Loop Temperature Control (cont)

- Effect of sampling period on performance index.



Quality of Control vs. Processing Cost (cont)

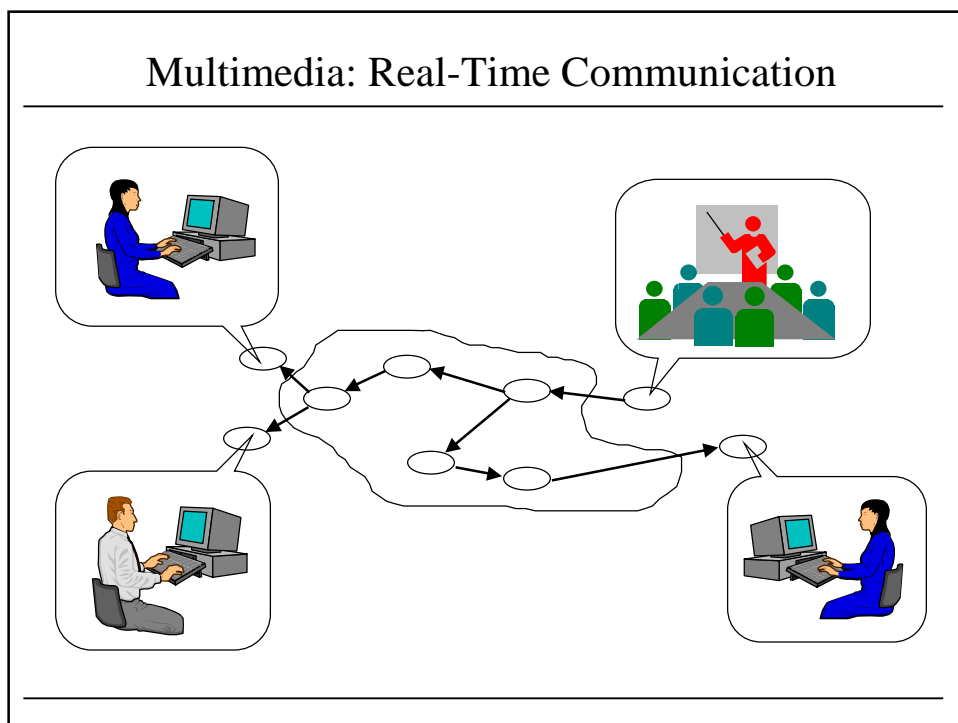
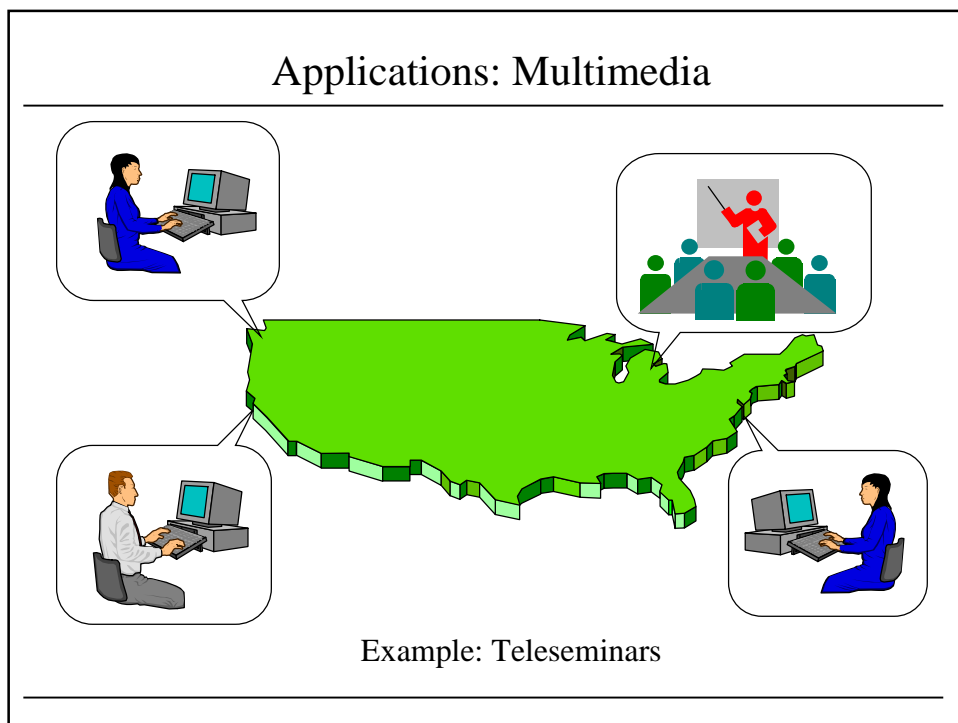
- Task frequencies must be determined to optimize the performance indices without overloading the available processing capabilities.
- Notation:

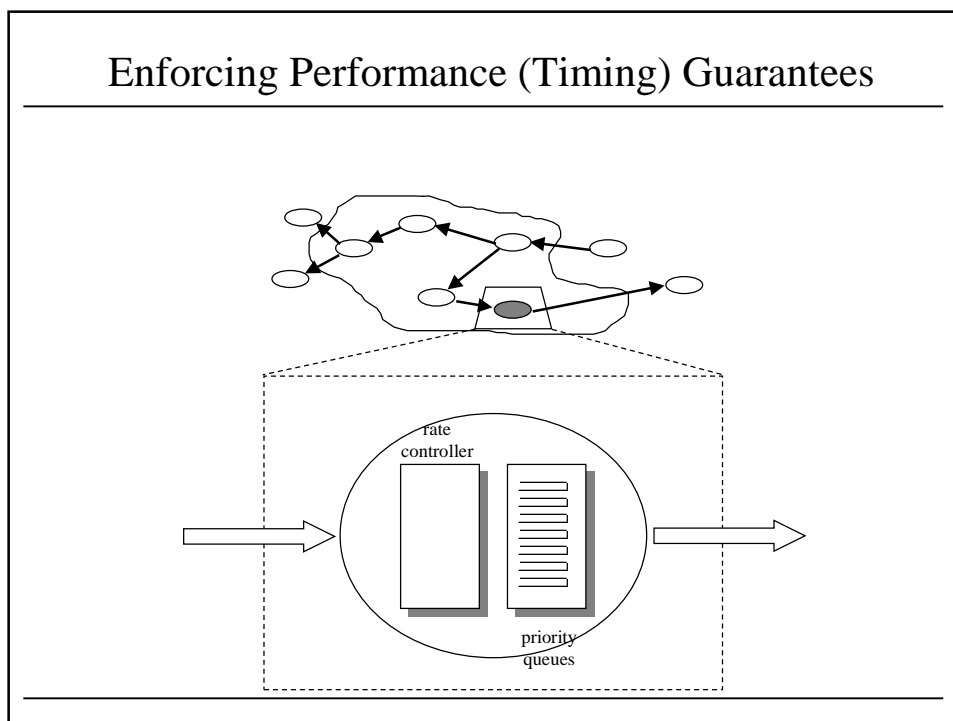
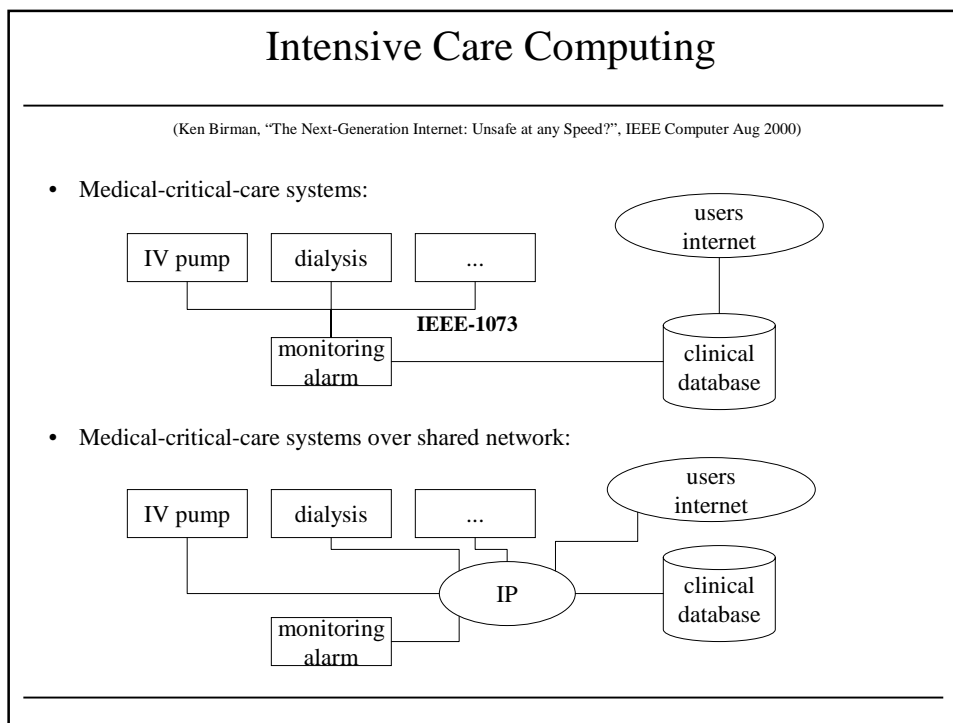
$$\Delta J^*(P) := J_D^*(P) - J^*$$

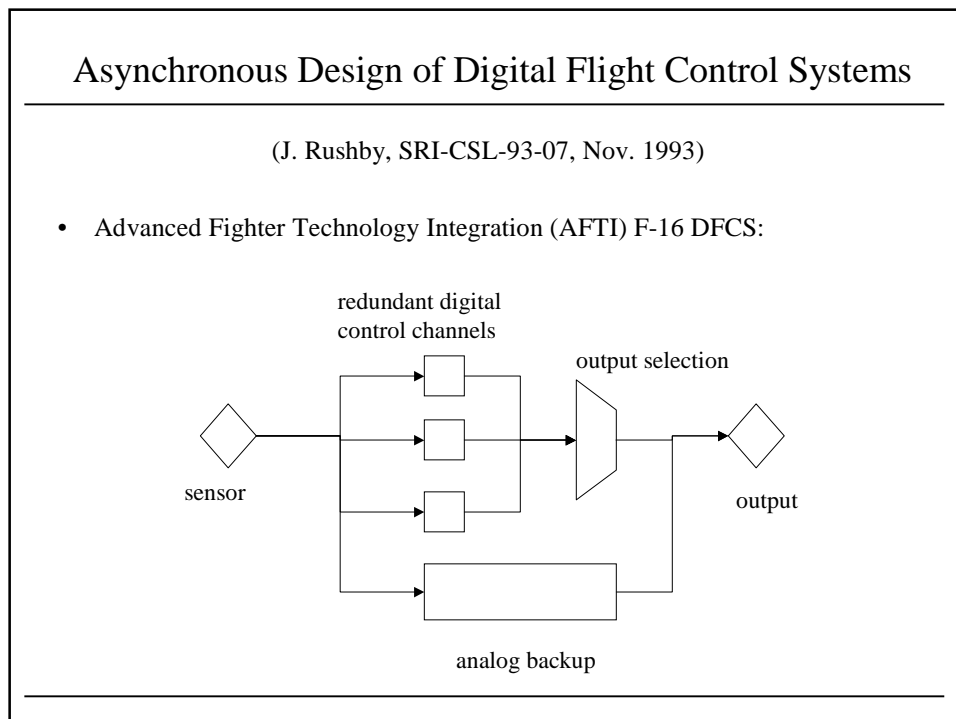
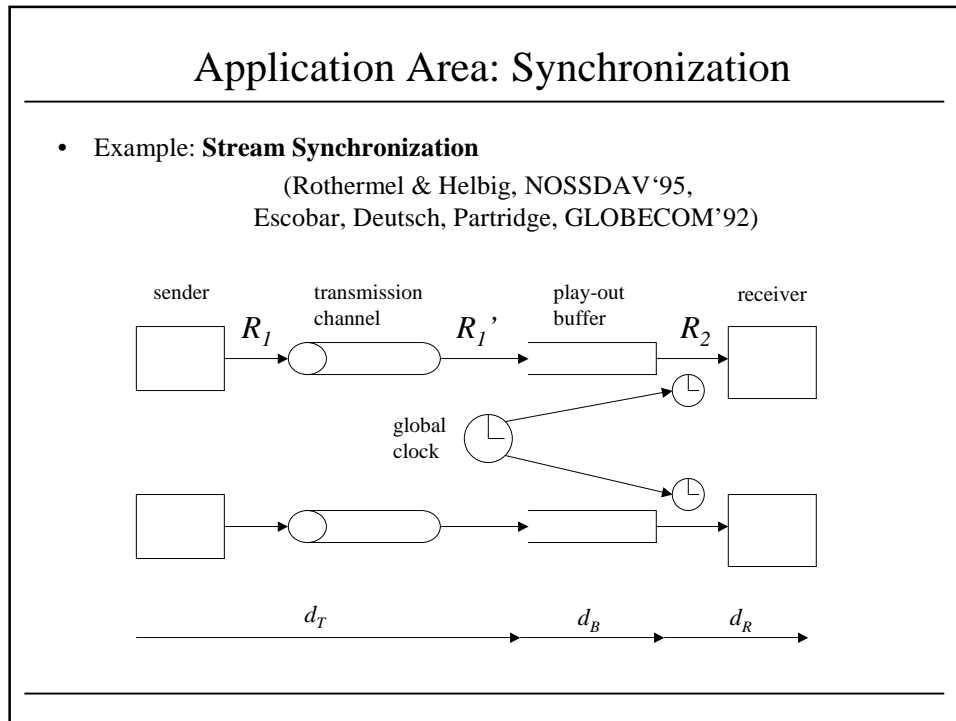
Optimization problem:

Given a set of tasks, T_1, \dots, T_n , with given $\Delta J_i^*(\bullet)$ and execution times C_i , find a set of periods P_i , such that

- $P_i \leq P_i^{max}$ // Maintain stability
- Minimize (maximize) $\sum_{i=1}^n \Delta J_i^*(P_i)$ // Optimize total performance index
- $\sum_{i=1}^n \frac{1}{P_i} C_i \leq U$ // CPU capacity constraints







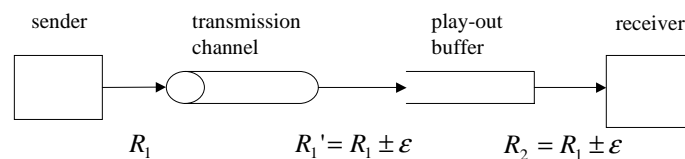
Asynchronous Design of Digital Flight Control Systems

“... The asynchronous design of the [AFTI-F16] DFCS introduced a random, unpredictable characteristic into the system. The system became untestable in that testing for each of the possible time relationships between the computers was impossible. This random time relationship was a major contributor to the flight test anomalies. Adversely affecting testability and having only postulated benefits, asynchronous operation of the DFCS demonstrated the need to avoid random, unpredictable, and uncompensated design characteristics.”

D. Mackall, flight-test engineer AFTI-F16 flight tests

Stream Synchronization: Issues

- Startup: Ensure that senders and receivers start transmission/presentation in synch.
- Buffer control: Keep size of play-out buffer in target area.
- Assume: underlying network gives **real-time guarantees**; a packet sent at time t is received during the interval $[t + D_{max} - J, t + D_{max}]$
 - D_{max} : **maximum delay** as guaranteed by the network
 - J : **maximum jitter**
- Benefits:
 - R_1' is bounded as a function of J .
 - If J is small enough, no synchronization necessary!



Real-Time vs. Non-Real-Time Systems

Q: What distinguishes RT systems from non-RT systems?

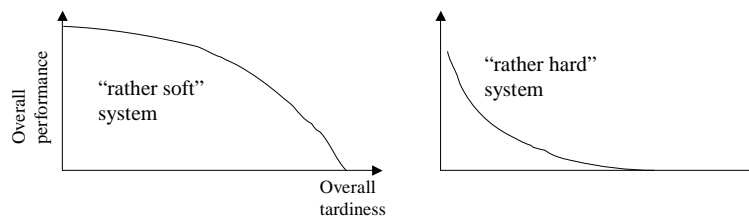
A: Timing constraints!

- Jobs and Processors:
 - **Job:** Unit of work executed by the system
 - **Processor:** Jobs require resource to execute (CPU, disk, network link)

No distinction necessary between types of processors!
- Timing constraints:
 - **Release Time:** time when job becomes available for execution
 - **Deadline:** time when execution must be completed
 - **Relative Deadline:** maximum response time

Hard vs. Soft Deadlines

- **Hard Deadline:** Late result may be a fatal flaw, of little use, or cause disastrous consequences
- **Soft Deadline:** Timely completion desirable. Late results useful to some degree
- Quantitative measure: Overall system performance as function of tardiness of jobs.



- Operational Definition: A job has a **hard** deadline whenever the system designer must prove that the job never misses its deadline.

Hard Real-Time Systems

Definition: A real-time system is **hard-real-time** when a large portion of the deadlines is hard.

- Examples:
 - Embedded systems
 - Recovery procedures in high-availability systems
- Does real-time mean fast ?
- Verification, certification: Why not use commercial OSs?
- Why requirements to meet deadlines 100% of the time?
 - Validation of probabilistic timing requirements.
 - Assessment of compound effect of missed deadlines with other factors.

Soft Real-Time Systems

Definition: A real-time system is a **soft-real-time** system when jobs have soft deadlines.

- Non-stringent timing requirements
 - on-line transaction system
 - telephone switches
- More stringent timing requirements
 - Stock price quotation system
- Stringent timing requirements
 - Multimedia
- Requirements often specified in probabilistic terms; validation is done by simulation, trial use.

