Assignment # 2
(Due February 15)

In this homework, I would like to let you practise some formal proof procedures for NP-completeness. Recall that to prove that a problem $Q$ is NP-complete, you need to prove that $Q$ is in NP and is also NP-hard. Please give formal and detailed proofs for both.

1. Vertex Cover  Let $G$ be a graph. A set $C$ of vertices in $G$ is a vertex cover if every edge in $G$ has at least one end in $C$.

Prove that the following problem is NP-complete:

**VERTEX COVER.** Given a graph $G$ and an integer $k$, decide if the graph $G$ has a vertex cover of at most $k$ vertices.

2. 0-1 Integer Programming  The 0-1 Integer Linear Programming problem is defined as follows:

Given integers $c_i$, $1 \leq i \leq n$, $b_j$, $1 \leq j \leq m$, $a_{pq}$, $1 \leq p \leq m$, $1 \leq q \leq n$, and $B$, decide if there exist $n$ values $x_i$, $1 \leq i \leq n$, where each $x_i$ is either 0 or 1, such that

$$c_1x_1 + c_2x_2 + \cdots + c_nx_n \leq B$$

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \geq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \geq b_2$$

$$\cdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \geq b_m$$

Prove that this problem is NP-complete.