

CPSC-637 Complexity Theory

Spring 2008

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Assignment # 2

(Due February 15)

In this homework, I would like to let you practise some formal proof procedures for NP-completeness. Recall that to prove that a problem Q is NP-complete, you need to prove that Q is in NP and is also NP-hard. Please give formal and detailed proofs for both.

1. Vertex Cover Let G be a graph. A set C of vertices in G is a *vertex cover* if every edge in G has at least one end in C .

Prove that the following problem is NP-complete:

VERTEX COVER. Given a graph G and an integer k , decide if the graph G has a vertex cover of at most k vertices.

2. 0-1 Integer Programming The 0-1 INTEGER LINEAR PROGRAMMING problem is defined as follows:

Given integers c_i , $1 \leq i \leq n$, b_j , $1 \leq j \leq m$, a_{pq} , $1 \leq p \leq m$, $1 \leq q \leq n$, and B , decide if there exist n values x_i , $1 \leq i \leq n$, where each x_i is either 0 or 1, such that

$$c_1x_1 + c_2x_2 + \cdots + c_nx_n \leq B$$

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \geq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \geq b_2$$

.....

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \geq b_m$$

Prove that this problem is NP-complete.