

## Overview

- Some more LISP stuff: user input, trace, cons, more setf, etc.
- Symbolic Differentiation:  
[q] does it need intelligence?
- Expression Simplification
- Programming Assignment (due 9/22/02, Sunday).

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## READ: User Input

READ: keyboard input from user

```
> (read)
hello
HELLO

> (if (equal (read) 'hello)
      'good
      'bad)
hello
GOOD
```

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## TRACE/UNTRACE: call tracing

```
> (trace fibo)
(FIBO)
> (fibo 4)
1> (FIBO 4)
2> (FIBO 3)
3> (FIBO 2)
<3 (FIBO 2)
3> (FIBO 1)
<3 (FIBO 1)
<2 (FIBO 3)
2> (FIBO 2)
<2 (FIBO 2)
<1 (FIBO 5)
```

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>

## List stuff

- CONS: append an atom and a list  
(cons 'a '(1 2 3)) -> (A 1 2 3)  
(cons '(a) '(1 2 3)) -> ((A) 1 2 3)
- APPEND: append two lists  
(append '(1 2) '(4 5)) -> (1 2 4 5)

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## Fun with SETF

Replace list element with SETF. Note: SETQ will not work!

```
>(setf b '(1 (2 3) 4))  
(1 (2 3) 4)
```

```
>(caaddr b)  
2
```

```
>(setf (caaddr b) 'abcdefg)  
ABCDEFG
```

```
>b  
(1 (ABCDEFG 3) 4)
```

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## Symbolic Differentiation

Basics: given variable  $x$ , functions  $f(x)$ ,  $g(x)$ , and constant (i.e. number)  $a$ :

1.

$$\frac{da}{dx} = 0, \frac{d(a \times x)}{dx} = a$$

2.

$$\frac{d(f + g)}{dx} = \frac{df}{dx} + \frac{dg}{dx}$$

3.

$$\frac{d(f \times g)}{dx} = \frac{df}{dx} \times g + f \times \frac{dg}{dx}$$

The operators can be extended to: binary minus (e.g.  $(- x 1)$ ), unary minus (e.g.  $(- x)$ ), division (e.g.  $(/ x 10)$ , etc.

## Symbolic Differentiation

Original concept and code borrowed from Gordon Novak's AI course at UTCS.

## Describing in LISP (I)

```
(deriv <expression> <variable>)
```

1.

$$\frac{da}{dx} = 0, \frac{d(a \times x)}{dx} = a$$

```
(deriv '10 'x) -> 0
```

```
(deriv '(* 10 x) 'x) -> 10
```

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## Describing in LISP (II)

```
(deriv <expression> <variable>)
```

1.

$$\frac{d(f + g)}{dx} = \frac{df}{dx} + \frac{dg}{dx}$$

```
(deriv '(+ (* x 10) (+ 25 x)) 'x)
== (list
    '+
    (deriv '(* x 10) 'x)
    (deriv '(+ 25 x))
)
```

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## DERIV: the core function

Pseudo code (basically a recursion):

```
(defun deriv (expression var) BODY)
```

1. if expression is the same as var return 1
2. if expression is a number return 0
3. if (first expression) is '+, return
 

```
'(+ (deriv (second expression) var)
      (deriv (third expression) var))
```
4. and so on.

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## Describing in LISP (III)

```
(deriv <expression> <variable>)
```

1.

$$\frac{d(f \times g)}{dx} = \frac{df}{dx} \times g + f \times \frac{dg}{dx}$$

```
(deriv '(* (+ 14 x) (* x 17)) 'x)
== (list
    '+
    (list '* (deriv '(+ 14 x) 'x) '(* x 17))
    (list '* '(+ 14 x) (deriv '(* x 17)))
)
```

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## Main Function: DERIV

You can make separate functions for each operator:

```
(defun deriv (expr var)
  (if (atom expr)
      (if (equal expr var) 1 0)
      (cond
        ((eq '+ (first expr)) ; PLUS
         (derivplus expr var))
        ((eq '* (first expr)) ; MULT
         (derivmult expr var))
        (t ; Invalid
         (error "Invalid Expression!"))))
)
```

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## Calling DERIV from DERIVPLUS

Then, you can call `deriv` from `derivplus`, etc.

```
(defun derivplus (expr var)
  (list '+
        (deriv (second expr) var)
        (deriv (third expr) var)
  )
)
```

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## SPLUS: Simplify (+ x Y)

```
(defun splus (x Y)
  (if (numberp x)
      (if (numberp Y)
          (+ x Y)
          (if (zerop x)
              Y
              (list '+ x Y)
          )
      )
      (if (and (numberp Y) (zerop Y))
          x
          (list '+ x Y)
      )
  )
)
```

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## Expression Simplification

Problem: a lot of nested expression containing

```
( * 1 x ) ( * x 1 ) ( + 0 x ) ( + x 0 ) ( + 3 4 ) ...
which are just x, x, x, x, x, and 7.
```

Use simplification rules:

1. `( + <number> <number> )`: return the evaluated value
2. `( * <number> <number> )`: return the evaluated value
3. `( + 0 <expr> )` `( + <expr> 0 )`: return `<expr>`
4. `( * 1 <expr> )` `( * <expr> 1 )`: return `<expr>`
5. `( - ( - <expr> ) )`: return `<expr>`

HINT: look at the raw result and see what can be reduced.

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## Programming Assignment 1

1. Implement `deriv` to support:  
addition, subtraction, unary minus, multiplication, and division.  
→ HINT: use slide 12 as a skeleton.
2. Implement simplification routines `splus` etc. for all operators  
and integrate it into `derivplus`, etc.  
→ HINT: Integrate code in slide 15 into code in slide 13.
3. Implement a function  
`(deriv-val <expr> <var> <value>)`  
to evaluate the final expression where the number `<value>`  
replaces the symbol `<var>`.  
→ HINT: Use the `eval` function to recursively evaluate.
4. Write a separate `(simplify <expr>)` function using  
`splus`, etc.

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## Programming Assignment 1: other conditions

All operators are either binary or unary:

i.e. expressions like  $( + 1 2 3 4 5 )$  do not need to be supported. Only those in the form of  $( + 1 2 )$  or  $( - 5 )$  are expected to be used.

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## Programming Assignment 1: Example Inputs and Outputs

1. `(deriv '( * ( + x 4 ) ( + x 5 ) ) 'x)`  
→ `( + ( + X 4 ) ( + X 5 ) )`
2. `(deriv '( / ( + x 1 ) x ) 'x)`  
→ `( / ( - X ( + X 1 ) ) ( * X X ) )`
3. `(deriv-val '( * ( + x 4 ) ( + x 5 ) ) 'x 10)`  
→ `29`
4. `(deriv-val '( / ( + x 1 ) x ) 'x 20)`  
→ `-1/400`

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## Programming Assignment 1: Required Material

Use the exact filename as shown below (in **bold**).

- Program code (**deriv.lsp**): put it in a single text file.
  - Ample indentation and documentation is required.
- Documentation (**README**): user manual
- Sample inputs and outputs (include in **README**)
  - 10 non-trivial (4 or more terms) examples should be given.
- Grading criteria:
  - README, test cases, comments, readability: 30%
  - deriv (with basic simplification) : 40%
  - deriv-val: 10%
  - simplify: 20%

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## Programming Assignment 1: Submission

- Use the unix `turnin` command. The folder name is 625-600. Turn in a single tar file (or `tar.gz`). Run `man turnin` to find out how to use it.
- Submission deadline is 9/22/02 Sunday midnight (23:59:59).
- Late policy: No late submissions allowed. Submit whatever you have.
- Only send plain text ASCII files. **Do not send MS-Word documents or other formatted text.**

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## **Next Time: Search Methods**

- Chapter 3
- Required: sections 3.3–3.7.
- Other sections are optional.