Trajectory Tracking and Balance Stabilization Control of Autonomous Motorcycles

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***Abstract*— In this paper, we present a trajectory tracking control algorithm for an autonomous motorcycle for the DARPA Grand Challenge. The mathematical dynamic model of the autonomous motorcycle is based on the existing work in modeling a bicycle and is extended to capture the steering effect on self-stabilization. The trajectory tracking control is designed using an external/internal model decomposition approach, and the motorcycle balancing is developed by the nonlinear control methods. The motorcycle balancing is guaranteed by the system internal equilibria calculation and by the trajectory and system dynamics requirements. The motion planning algorithms are based on the fused global positioning systems (GPS) and on-board computer vision systems information. The proposed control system is validated by numerical simulations, which are based on a real prototype motorcycle system.**

I. Introduction

Single-track vehicles, such as motorcycles and bicycles, provide flexible maneuverability and deployments. Particularly for off-road environments such as deserts, mountains, and forests, single-track vehicles have a superior performance in comparison to double-track vehicles. Moreover, the light weight of motorcycles also provides attractive properties such as high energy efficiency and fast acceleration. Motivated by this observation, an autonomous motorcycle platform is employed, designed, and built for the DARPA Grand Challenge competition[1], a racing competition of autonomous ground vehicles on off-road trails in the desert in southern California.

In this paper, we present a trajectory tracking and balancing control design for the autonomous motorcycle. The mathematical model and control algorithms of the autonomous motorcycle are developed and extended on the existing research results in [1]. This mathematical model for motorcycles has several attractive features when compared with other models in the literature. First, multibody dynamics models in the literature are very complex and are not suitable for the control design system. Simple inverted pendulum models such as the one used in [2] cannot capture all of the dynamics characteristics of the motorcycle system, such as under-actuation and nonholonomic constraints. The model used in this paper has some simplifications from the multibody dynamics model, but also keeps the nonholonomic constraint properties of the system. Therefore, the model provides enough accuracy of the system and is manageable for the purposes of control system design. We extend this model and take considerations of motorcycle trail and caster angle, which are important geometric parameters in motorcycle dynamics and are neglected in [1]. The control algorithms guarantee the asymptotic tracking of the desired trajectory given by the motion planning module. The motion planning is based on the fusion of an on-board global positioning system (GPS) and a computer vision system. We demonstrate the integrated motion planning, trajectory tracking, and balancing control of the autonomous motorcycle through numerical simulations. The experimental testing of the proposed algorithms on a real motorcycle prototype is ongoing under the current research.

The rest of this paper is organized as follows. We begin with a review of the related work in section II. In section III, we discuss the mathematical model for the motorcycle dynamics. Section IV presents the control system design for a trajectory following and balancing stabilization control. Motion planning for the motorcycle system is discussed in section V. Numerical simulation results of the autonomous motorcycle system are presented in section VI. Finally, the conclusion remarks and future direction are given in section VII.

II. Related Work

Modeling and control of a bicycle or a motorcycle is a challenging task. There are quite a few research works that study bicycle or motorcycle stability and dynamics. Sharp [3] discusses mathematical models of a motorcycle with a rider using Lagrange's equation. Sharp [4] studies the multibody dynamics motorcycle model using dynamics simulation packages AutoSim and Matlab. Independently, Cossalter and Lot [5], Cossalter [6], and Kessler [7] study a nonlinear multibody dynamics model of a motorcycle. The model developed in [5] is very comprehensive and contains front and rear chassis, steering system, suspension, and tires. The model was implemented in a simulation package called *FastBike* for the purposes of real-time simulations. The *FastBike* simulation results are also compared with experimental data in [5]. Motorcycle tire models are developed in [8], [9]. Modal analysis and steering characteristics of a motorcycle are discussed in [4], [10] based on the multibody dynamics developed by the authors, respectively. Experimental study of the motorcycle handling is compared with the mathematical dynamics model of a motorcycle with the rider in [11].

Bicycles share many similar properties with motorcycles. Jones [12] studies the stability properties of a bicycle. In [12],

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[1] http://www.darpa.mil/grandchallenge/

the non-significance of the gyroscopic effect of the front wheel has been verified experimentally. The author proposes that the geometry of the bicycle (trail) is the most important factor for the bicycle stability. Lowell and McKell [13] analyze a simplified bicycle model to explain the stability. Fajans [14] discusses the steering of a bicycle or a motorcycle using a simple mathematical model and some numerical simulations. The simplified models in [13], [14] are derived from Newtonian force and momentum balance principles, and balancing by the rider is the main concern of these studies. Recently, Åström *et al.* [15] review and discuss bicycle dynamics from the perspective of automatic controls.

The concept of an autonomous bicycle has been proposed by several researchers. Getz and Marsden [16] utilize an input/output linearization method to design a trajectory tracking controller for an autonomous bicycle. Under such a trajectory tracking control, the bicycle is balanced to its internal equilibria. Simulation study has been carried out to show the effectiveness of the proposed control. In [17], an autonomous bicycle is designed and balanced using gyroscopic actuators. The controller is designed based on a linearized bicycle model, and the experiments are not successful for tracking a given trajectory. Tanaka and Murakami [2] use a simplified inverted pendulum model for bicycle balancing. A proportional derivative (PD) controller with a disturbance observer is designed to balance the bicycle in an indoor laboratory setup. Balancing is the main concern and no trajectory tracking control is discussed in [2]. In this paper, we extend the model and control algorithms in [1] and take considerations of motorcycle trail and caster angle. We also integrate the control algorithms with the GPS/vision-based motion planning system to design a control system, which guarantees that the motorcycle follows the off-road tracks.

III. Motorcycle Dynamics

Nomenclature

m	Motorcycle mass
g	Gravity constant $g = 9.8\ m/s^2$
L	Motorcycle wheelbase
h	Height of the motorcycle center of mass
b	Horizontal distance between rear wheel contact point and the motorcycle center of mass
R	Motorcycle turning radius
η	Caster angle
Δ	Motorcycle trail
θ	Roll (camber) angle of the motorcycle frame (rear wheel)
ψ	Yaw angle of the motorcycle frame (rear wheel)
ϕ	Steering angle
β	Front wheel direction angle (effective steering angle)

We consider the motorcycle as a point mass with two wheel contacts with the ground. For simplicity, we consider a rear-wheel driving and front-wheel steering motorcycle with the following assumptions.

Assumption 1: Motorcycle system assumptions.

1. The front and rear wheel tires are thin, and we do not consider the thickness of the tire.
2. No slip angle is considered for both front and rear tires, i.e. the wheel speed is aligned with wheel planes.
3. The moments of mass of the front and rear wheels are neglected.
4. The moments of mass of the motorcycle are neglected, i.e. the motorcycle is considered as a point mass at the mass center.
5. Only rear wheel longitudinal force has been considered. The front wheel longitudinal force is neglected.
6. The motorcycle is moving in a flat plane XOY, and no vertical motion is considered.

Fig. 1 shows a schematic diagram of an autonomous motorcycle. Two variables are considered for the control input: the steering angle ϕ and the rear-wheel driving torque τ.

Consider that the generalized coordinates of the motorcycle are $(x, y, \psi, \theta, \sigma)$, where (x, y) are the Cartesian coordinates of the rear wheel contact point C_2. For roll angle θ, we define tilting right from the vertical line is positive and for steering angle ϕ, turning left is the positive direction. Denote the radius of the trajectory of the rear-wheel contact point C_2 as R and the trajectory curvature is defined as σ

$$\sigma = \frac{1}{R} = \frac{\tan\beta}{L}\,. \tag{1}$$

From the equation above, we take the time derivative and obtain

$$\dot{\beta} = \frac{L}{1+\tan^2\beta}\dot{\sigma} = \frac{L}{1+L^2\sigma^2}\dot{\sigma}. \tag{2}$$

From the geometry of the front wheel steering mechanism, we can find the following relationship among the steering angle ϕ, direction angle β, roll angle θ, and caster angle η,

$$\tan\beta\cos\theta = \tan\phi\sin\eta\,. \tag{3}$$

From Fig. 1(b), we can calculate the relationship between (x, y) and yaw angle ψ as

$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}\cos\psi & -\sin\psi\\ \sin\psi & \cos\psi\end{bmatrix}\begin{bmatrix}v_r\\ v_\perp\end{bmatrix}, \tag{4}$$

where v_r is the rear wheel longitudinal velocity and $v_\perp$ is the lateral velocity. The nonholonomic constraint of the rear wheel implies the following equalities [1].

$$\begin{cases}\dot{\psi} = \sigma v_r\\ v_\perp = 0\,.\end{cases} \tag{5}$$

In [1], the author assumes that the steering axis is vertical. This assumption simplifies the motorcycle dynamics and neglects a significant geometric stabilization mechanism, "motorcycle trail" (denoted as Δ in Fig. 1(a)) discussed in [6], [12]–[14]. The resulting model of the motorcycle dynamics cannot capture the influence of the steering angle ϕ on the roll dynamics when $v_r = 0$. Namely, one cannot use steering to stabilize the motorcycle when it is still.

In order to capture the stabilization mechanism of the motorcycle at a low moving velocity, we can modify the constructed motorcycle Lagrangian L_c by considering the

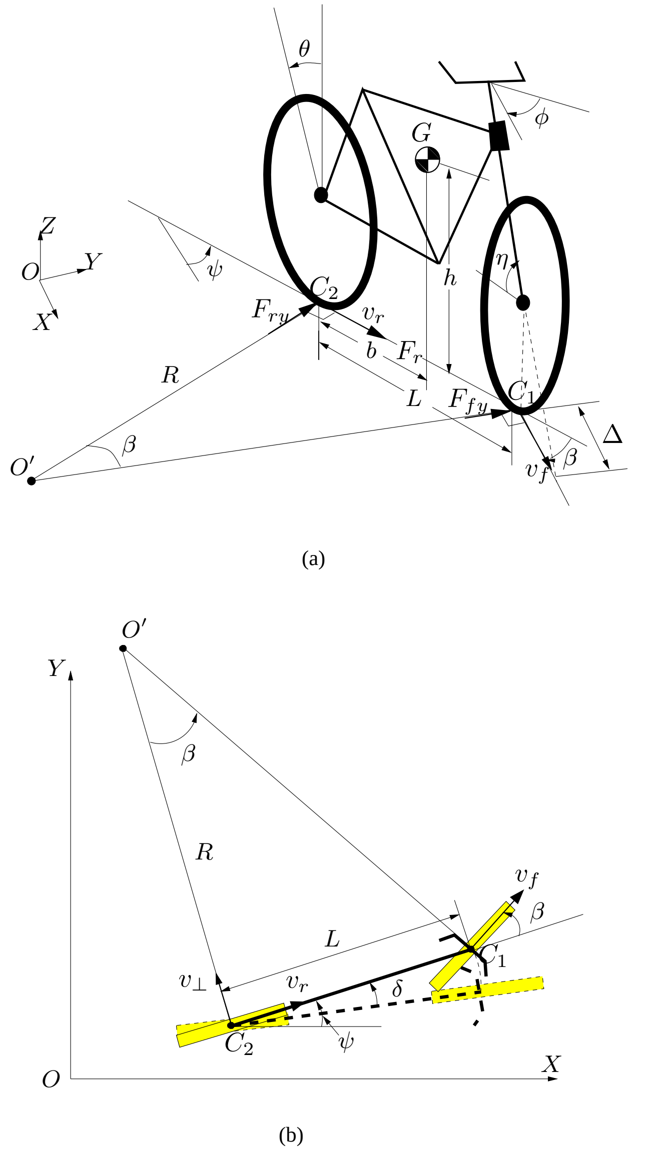


Fig. 1. A kinematic diagram of the motorcycle dynamics. (a) Schematic view of a motorcycle on a plane, (b) top-view of the motorcycle.

mass center gravity change due to steering. The change of the mass center position due to the steering action can be calculated by the estimate of the motorcycle frame rotation angle δ in the horizontal plane XOY (as shown in Fig. 1(b)). From [6], δ can be estimated approximately as

$$\delta = \frac{\Delta \sin\eta}{L}\beta\,. \tag{6}$$

The reduction of the mass center height can be calculated as

$$h_{G\Delta} = \delta b \sin\theta = \frac{b\Delta \sin\eta}{L}\beta\sin\theta \approx b\Delta\sigma\sin\eta\sin\theta\,. \tag{7}$$

Therefore, we can derive the constrained motorcycle Lagrangian as

$$\begin{aligned} L_c &= -mg\left(h\cos\theta - b\Delta\sigma\sin\eta\sin\theta\right) + \frac{1}{2}J_s\dot{\phi}^2 + \frac{1}{2}mv_G^2 \\ &= -mg\left(h\cos\theta - b\Delta\sigma\sin\eta\sin\theta\right) + \frac{1}{2}J_s\dot{\phi}^2 + \\ &\quad \frac{1}{2}m\big[(v_r + h\sigma v_r\sin\theta)^2 + h^2\dot{\theta}^2\sin^2\theta + \\ &\quad (b\sigma v_r - h\dot{\theta}\cos\theta)^2\big]\,, \end{aligned} \tag{8}$$

where J_s is the moment of inertia of the steering mechanism, and v_G is the translational velocity of the motorcycle mass center.

Following the same derivation in [1] with the modified Lagrangian (8), we can obtain the motorcycle dynamics as

$$\dot{\sigma} = \omega_\sigma \tag{9}$$

$$\mathbf{M}\begin{bmatrix}\ddot{\theta}\\ \ddot{v}_r\end{bmatrix} = \mathbf{K} + \mathbf{B}\begin{bmatrix}\omega_\sigma\\ F_r\end{bmatrix}, \tag{10}$$

where ω_σ is the virtual steering angular velocity input, F_r is the longitudinal tracking force at the rear wheel and

$$\mathbf{M} = \begin{bmatrix} h^2 & -bh\sigma\cos\theta \\ -bh\sigma\cos\theta & b^2\sigma^2 + (1+h\sigma\sin\theta)^2 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} bhv_r\cos\theta & 0 \\ -\left[b^2\sigma + h\sin\theta(1+h\sigma\sin\theta)\right]v_r & \frac{1}{m} \end{bmatrix}$$

and

$$\mathbf{K} = \begin{bmatrix} g(h\sin\theta + b\Delta\sigma\sin\eta\cos\theta) + (1+h\sigma\sin\theta)h\sigma v_r^2\cos\theta \\ -2h\sigma v_r\dot{\theta}(1+h\sigma\sin\theta)\cos\theta - bh\sigma\dot{\theta}^2\sin\theta \end{bmatrix}.$$

Remark 1: Eq. (9) is obtained through the variable substitution of steering angle ϕ with the generalized coordinate σ. From Eq. (3) and a small angle approximation, we obtain

$$\dot{\beta} = \dot{\phi}\sin\eta = \frac{L}{1+L^2\sigma^2}\dot{\sigma}$$

and

$$\frac{1}{2}J_s\dot{\phi}^2 = \frac{1}{2}\left(J_s\frac{L^2}{(1+L^2\sigma^2)^2\sin^2\eta}\right)\dot{\sigma}^2 = \frac{1}{2}J(\sigma)\dot{\sigma}^2\,,$$

where $J(\sigma)$ is the generalized moment of inertia of the steering mechanism. Using the constrained Lagrangian L_c defined by Eq. (8), we can obtain the dynamics of σ as

$$J(\sigma)\ddot{\sigma} - \frac{1}{2}\frac{dJ(\sigma)}{d\sigma}\dot{\sigma}^2 - mgb\Delta\sin\eta\cos\theta = \tau_s - F_{fy}\Delta\cos\eta\,, \tag{11}$$

where τ_s is the steering torque by the steering motor and F_f is the lateral friction force at the front wheel (the lateral friction force F_{ry} at the rear wheel is neglected here). Using the assumption of the fast actuation of the steering systems and the small moment of inertia of the front-wheel steering mechanism, we simplify the dynamics of σ (11) into Eq. (9). Due to the relationship (2), the steering input ω_σ can be considered as the desired angular velocity of the steering. For a realistic model, we can consider a first-order approximation modeling of the steering system instead of the pure integral form given in Eq. (9).

Therefore, the control inputs in Eqs. (9) and (10) are the virtual steering control ω_σ and the rear-wheel force F_r.

IV. CONTROL SYSTEM DESIGN

The control of an autonomous motorcycle has to satisfy two specifications. First, the motorcycle has to be balanced so it will not fall down. Second, the motorcycle must follow the desired trajectory. The desired trajectory is normally generated by a motion planning scheme. In this section, we assume that the desired trajectory $(x_d(t), y_d(t))$ is given, and in the next section, we will briefly describe how to obtain such a desired trajectory based on GPS and vision systems in real-time. The control system design follows the external/internal convertible form approach proposed in [1]. We consider a more realistic motorcycle model given by Eqs. (9) and (10) and also design a balance stabilization controller when the vehicle is still. In the following, we first describe the control design for a moving vehicle. Then we discuss a balance stabilization control when the motorcycle is stationary.

A. Trajectory tracking control

First, we rewrite the system dynamics (9) and (10) into an external/internal convertible form [1]. Using the input transformation

$$F_r = B_{22}^{-1} M_{22} \left[-M_{22}^{-1} \left(-M_{21}\ddot{\theta} + K_2 + B_{21}\omega_\sigma \right) + a_r \right],$$

Eq. (10) becomes

$$\begin{cases} \dot{v}_r = a_r \\ M_{11}\ddot{\theta} = K_1 - M_{12}a_r + B_{11}\omega_\sigma , \end{cases} \tag{12}$$

where M_{ij} (B_{ij}) is the element of matrix $\mathbf{M}$ ($\mathbf{B}$) and K_i is the ith element of vector $\mathbf{K}$. In order to connect the outputs (x, y) of the rear wheel contact point to the inputs, we differentiate the outputs (x, y) three times and obtain

$$\begin{bmatrix} x^{(3)} \\ y^{(3)} \end{bmatrix} = \begin{bmatrix} -2\dot{v}_r\dot{\psi}\sin\psi - v_r\dot{\psi}^2\cos\psi \\ 2\dot{v}_r\dot{\psi}\cos\psi - v_r\dot{\psi}^2\sin\psi \end{bmatrix} + \begin{bmatrix} \cos\psi & -v_r\sin\psi \\ \sin\psi & v_r\cos\psi \end{bmatrix} \begin{bmatrix} \ddot{v}_r \\ \ddot{\psi} \end{bmatrix}.$$

Define the new input variables

$$u_r = \ddot{v}_r = \dot{a}_r, \quad u_\psi = \ddot{\psi} = \dot{v}_r\sigma + v_r\dot{\sigma} = a_r\sigma + v_r\omega_\sigma . \tag{13}$$

With the new inputs u_r and u_ψ, we can rewrite the motorcycle dynamics (9) and (12) in terms of (x, y, θ) as

$$\begin{bmatrix} x^{(3)} \\ y^{(3)} \end{bmatrix} = \begin{bmatrix} -2\dot{v}_r\sin\psi - v_r\dot{\psi}\cos\psi \\ 2\dot{v}_r\cos\psi - v_r\dot{\psi}\sin\psi \end{bmatrix}\dot{\psi} + \begin{bmatrix} \cos\psi & -v_r\sin\psi \\ \sin\psi & v_r\cos\psi \end{bmatrix} \begin{bmatrix} u_r \\ u_\psi \end{bmatrix} \tag{14a}$$

$$\ddot{\theta} = \frac{g}{h}\left(\sin\theta + \frac{b\Delta\dot{\psi}}{hv_r}\cos\theta\right) + \frac{1}{h}\left(1 + \frac{h\dot{\psi}}{v_r}\sin\theta\right)\dot{\psi}v_r\cos\theta + \frac{b}{h}u_\psi\cos\theta . \tag{14b}$$

We can design an external system controller to track the desired trajectory $(x_d(t), y_d(t))$ asymptotically as

$$\begin{bmatrix} u_{r0} \\ u_{\psi 0} \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi \\ -\frac{\sin\psi}{v_r} & \frac{\cos\psi}{v_r} \end{bmatrix} \left(- \begin{bmatrix} -2\dot{v}_r\sin\psi - v_r\dot{\psi}\cos\psi \\ 2\dot{v}_r\cos\psi - v_r\dot{\psi}\sin\psi \end{bmatrix}\dot{\psi} + \begin{bmatrix} u_x \\ u_y \end{bmatrix} \right), \tag{15}$$

where

$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} = \begin{bmatrix} x_d^{(3)} \\ y_d^{(3)} \end{bmatrix} - \sum_{i=1}^{3}\gamma_i \begin{bmatrix} x^{(i-1)} - x_d^{(i-1)} \\ y^{(i-1)} - y_d^{(i-1)} \end{bmatrix}. \tag{16}$$

The constants γ_i, $i = 1, 2, 3$, are chosen such that the polynomial equation $s^3 + \gamma_3 s^2 + \gamma_2 s + \gamma_1 = 0$ is Hurwitz.

By external control (15), the internal (roll) equilibrium angle is given by substituting $u_{\psi 0}$ into the dynamics of θ in Eq. (14), and therefore we obtain that the roll angle equilibrium θ_e must satisfy the following algebraic equation

$$\frac{g}{h}\left(\tan\theta_e + \frac{b\Delta\dot{\psi}}{hv_r}\sin\eta\right) + \frac{1}{h}\left(1 + \frac{h\dot{\psi}}{v_r}\sin\theta_e\right)\dot{\psi}v_r + \frac{b}{h}u_{\psi 0} = 0 . \tag{17}$$

Then the final control system is designed as

$$u_r = u_{r0} \tag{18}$$

$$u_\psi = \left(\frac{b}{h}\cos\theta\right)^{-1}\left[-\frac{g}{h}\left(\sin\theta + \frac{b\Delta\dot{\psi}}{hv_r}\sin\eta\cos\theta\right) - \frac{1}{h}\left(1 + \frac{h\dot{\psi}}{v_r}\sin\theta\right)\dot{\psi}v_r\cos\theta + v_\psi \right], \tag{19}$$

where

$$v_\psi = \ddot{\theta}_e - \beta_2(\dot{\theta} - \dot{\theta}_e) - \beta_1(\theta - \theta_e)$$

and constants β_1 and β_2 are chosen such that the polynomial equation $s^2 + \beta_2 s + \beta_1 = 0$ is Hurwitz. The calculation of $\ddot{\theta}_e$ and $\dot{\theta}_e$ in the above equation is estimated by dynamic inversion of θ_e. Details can be found in [1].

The equilibrium of the internal state, roll angle θ_e, is calculated by the implicit function in Eq. (17). However, the motorcycle dynamics we considered here do not include the lateral friction forces, and therefore solution of Eq. (17) cannot guarantee $\theta_e \leq \theta_{max}$, where θ_{max} is the maximum roll angle allowed at a given motorcycle state. The maximum roll angle θ_{max} is directly related to the motorcycle tire/road characteristics and dynamics. We can estimate θ_{max} approximately as follows. When the motorcycle is running under normal conditions with roll angle θ, the tire/road contact points, C_1 and C_2, are not sliding on the ground. The friction forces F_{fy} and F_{ry} balance the centrifugal force. From a balance relationship of static forces and moments, we can calculate the roll angle as

$$\theta = \tan^{-1}\left(\frac{v_r^2}{gR}\right).$$

The lateral friction force F_y has been studied in [6], [9]. In this study we only consider the static lateral force with the tire slip angle $\lambda = 0$. The lateral friction force can be modeled as

$$F_y = k_\lambda \lambda + k_\theta \theta = k_\theta \theta \,,$$

where k_λ and k_θ are the tire sideslip stiffness and camber stiffness coefficients, respectively. Assuming the static tire/road friction coefficient is μ_s, then we can estimate θ_{max} as

$$\theta_{max} = \frac{\mu_s}{k_\theta} \,. \tag{20}$$

For motorcycle balance stabilization, we have to maintain $\theta \leq \theta_{max}$ for all time. We can design the desired trajectory to satisfy $\theta_e \leq \theta_{max}$ and the tracking control given by Eqs. (18), and (19) will maintain that the roll angle converges to the internal equilibrium manifold.

B. Balance stabilization at zero velocity

When the motorcycle is still, i.e. $v_r = 0$, the dynamics are reduced to

$$\begin{cases} \dot{\sigma} = \omega_\sigma \\ \ddot{\theta} = \frac{g}{h}\left(\sin\theta + \frac{b\Delta \sin\eta}{h}\sigma\cos\theta\right) \,. \end{cases} \tag{21}$$

We can design a robust sliding mode control for the roll angle θ as follows. Consider the input of the system (21) as the steering angle ϕ, which is related to σ by relationship (1) and (2). We can rewrite Eq (21) as

$$\ddot{\theta} = f_1(\theta) + f_2(\theta) u \,,$$

where $f_1(\theta) = \frac{g}{h}\sin\theta$, $f_2(\theta) = \frac{gb\Delta\sin\eta}{h^2 L}\cos\theta$, and $u = \tan\beta$.

For motorcycle stability, we have $|\theta| \leq \theta_{max}$, where θ_{max} is the known maximum stabilizing roll angle. Denote the measured roll angle as $\hat{\theta}$ and real roll angle θ. Define $\cos\theta_0 = \sqrt{\cos\theta_{max}\cos 0} = \sqrt{\cos\theta_{max}}$. Therefore, we can obtain

$$f_1(\theta) = \bar{f}_1(\hat{\theta}) + \delta f_1, \quad f_2(\theta) = \bar{f}_2 \cdot \delta f_2 \,, \tag{22}$$

where

$$\bar{f}_1(\hat{\theta}) = \frac{g}{h}\sin\hat{\theta}\,, \quad |\delta f_1| = \left|\frac{g}{h}\left(\sin\theta - \sin\hat{\theta}\right)\right| \leq \alpha = \frac{2g}{h}$$

and

$$\bar{f}_2 = \frac{gb\Delta\sin\eta}{h^2 L}\cos\theta_0\,, \quad \delta f_2 = \frac{\cos\hat{\theta}}{\cos\theta_0}\,.$$

Define $\beta_{min} = \frac{\cos\theta_{max}}{\cos\theta_0}$ and sliding surface $s = \dot{\epsilon} + \lambda\epsilon$, where $\epsilon = \theta - \theta_d$, and $\theta_d = 0$ is the desired roll angle. Consider the following virtual control algorithm

$$u_v = -\frac{1}{\bar{f}_2}\left(\lambda\dot{\hat{\theta}} + \bar{f}_1(\hat{\theta}) + Ks\right) \tag{23}$$

with

$$K = \frac{1}{\beta_{min}}\left[(1 - \beta_{min}|\lambda\dot{\hat{\theta}} + \bar{f}_1(\hat{\theta})| + \alpha + \zeta\right]$$

and $\zeta > 0$. It can be proven [18] that under the virtual control law (23)

$$s\dot{s} \leq -\zeta s^2 \,.$$

In order to smooth the actual input steering angle, we use a low-pass filter to smooth the virtual input u_v as

$$\tau_f \dot{u} + u = u_v \,,$$

where τ_f is the filter coefficient. It can be proven that the use of the low-pass filter will not affect the stability results [19].

V. Motorcycle Motion Planning

In section IV, we design the trajectory control algorithms to guide the motorcycle to follow the desired trajectory while achieving self-stability. The desired trajectory is provided by the on-board motion planning module. The equipped GPS cannot provide enough information for obstacle avoidance and vehicle navigation. We propose to fuse the computer vision and GPS signals to generate the motorcycle navigation trajectory in real-time. We briefly describe the combined vision/GPS navigation algorithms. More detailed information about such algorithms will be reported separately.

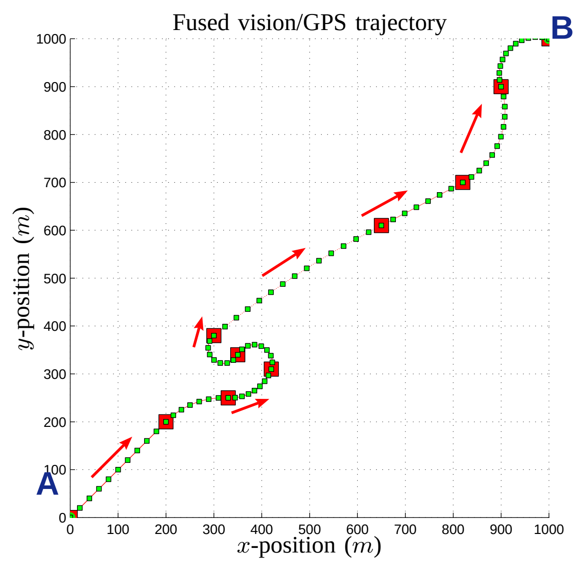


Fig. 2. An example of motorcycle trajectory from point A (with coordinates $(0, 0)$) to point B (with coordinates $(1000m, 1000m)$ (large red squares represent GPS waypoints and smaller green squares represent the fused vision/GPS target points).

As shown in Fig. 2, GPS signals only provide limited waypoint information about the unpaved motorcycle trail. In order to guarantee that the motorcycle follows such a trail, we use an on-board vision system to provide unpaved road information. The computer vision algorithms provide a set of vehicle direction candidates using a so-called V-space approach. By weighting these with GPS information, the algorithms provide a motion planning output to the lower-level motorcycle controller given in section IV. Fig. 3 shows an example of the fused GPS/vision direction output on an offroad trail in the desert in southern California.

The motion planning scheme generates the desired trajectory of the motorcycle for a fixed-period ΔT based on the current states, such as position, velocity, and other kinematics constraints. As shown in Fig. 4, for a small ΔT, the desired

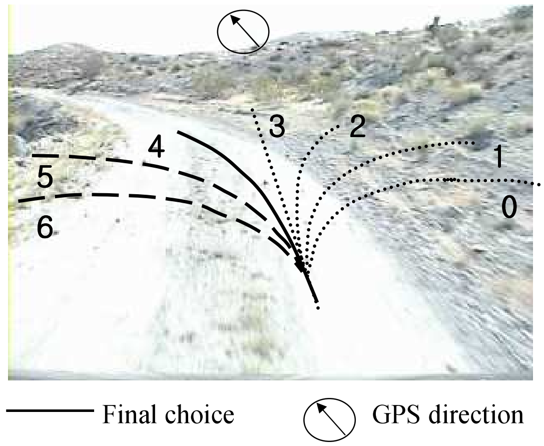


Fig. 3. An example of the fused GPS/vision direction output on an offroad trail in the desert. Dashed lines are candidate feasible trajectories from vision systems and the solid line is the final choice direction fused with GPS information.

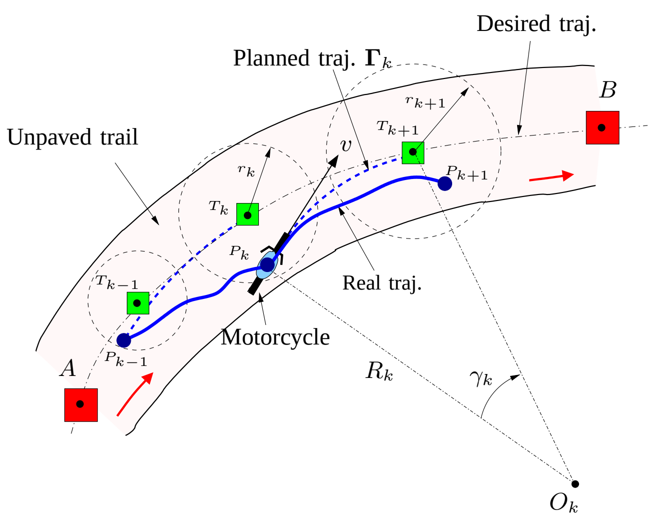


Fig. 4. A schematic of the vision/GPS motion planning.

trajectory can be approximated by a circular curve $\mathbf{\Gamma}_k$ that connects current motorcycle position p_k and the next target point T_{k+1}. A pre-defined radius r_k around the current target point T_k is used to eliminate T_k from the candidate set for the next target point if $|p_k - T_k| \leq r_k$. At motorcycle position p_k, the motion planning output can be represented by a triplet

$$\mathcal{T}_k(t) = \{v_k(t), R_k, d_k\}, \quad t \in [0, \Delta T), \tag{24}$$

where $v_k(t)$ is the desired velocity (magnitude) profile within the period $[0, \Delta T)$, R_k is the desired trajectory radius, and d_k is a binary direction variable to indicate on which side the trajectory $\mathbf{\Gamma}_k$ is located relative to the current velocity vector v. For example, $d_k = 1$ indicates that the desired trajectory $\mathbf{\Gamma}_k$ is on the right side of the current velocity direction v, and $d_k = -1$ if $\mathbf{\Gamma}_k$ is on the left side of v. The desired velocity profile v_k will be determined by the motorcycle kinematic constraints and also the turning radius R_k. Since the trajectory tracking controller is asymptotically stable, the real motorcycle position p_{k+1} at the end of each time period ΔT is close to the target point T_{k+1}. Then, we start the next motion planning period from location p_{k+1}.

VI. Simulation Results

In this section, we only show some simulation results of the integrated trajectory tracking control systems based on the prototype hardware systems. The GPS/vision-based motion planning algorithms have been implemented and tested on a two-wheeled mobile robot. The experimental results are reported separately. The proposed trajectory tracking control system is undergoing tests on real autonomous motorcycle systems with the motion planning module. We will report the experimental results once they are available.

A. Autonomous motorcycle

Fig. 5 shows the autonomous motorcycle built by the Blue team for the DARPA Grand Challenge. The motorcycle is built on a commercial system with significant modifications. The motorcycle is equipped with sensors, actuators and several computing systems. The on-board sensing devices have the capability to measure the motorcycle's linear velocity and acceleration, roll angle, roll rate and acceleration, steering angle, and global position etc. in real time. The controlled actuations are traction/braking and front-wheel steering. Table I shows the parameter estimations of the motorcycle shown in Fig. 5. We use these parameters in the simulation.

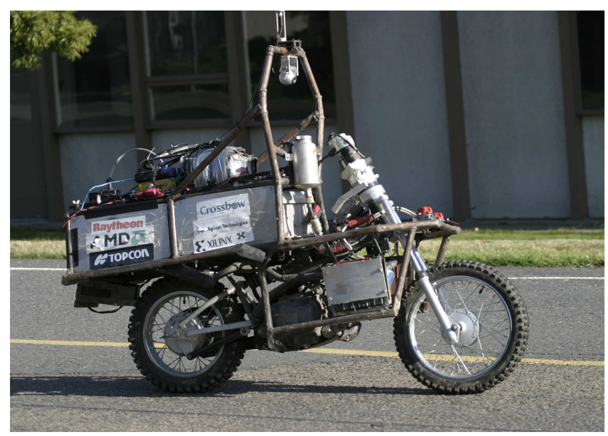


Fig. 5. A picture of the Blue team autonomous motorcycle.

TABLE I

Geometric parameters of the autonomous motorcycle

Variables	L (m)	b (m)	h (m)	Δ (m)	η (deg.)
Values	1.2	0.8	0.6	0.2	70

B. Model calibration and simulation results

1) Trajectory tracking: Fig. 6 shows the simulation results of the trajectory-following algorithms for the example trail given in Fig. 2. The motion planning period is updated every 3 s, i.e. $\Delta T = 3$ s. The desired velocity $v_k(t)$ in each planning period ΔT is simply calculated as a function of trajectory

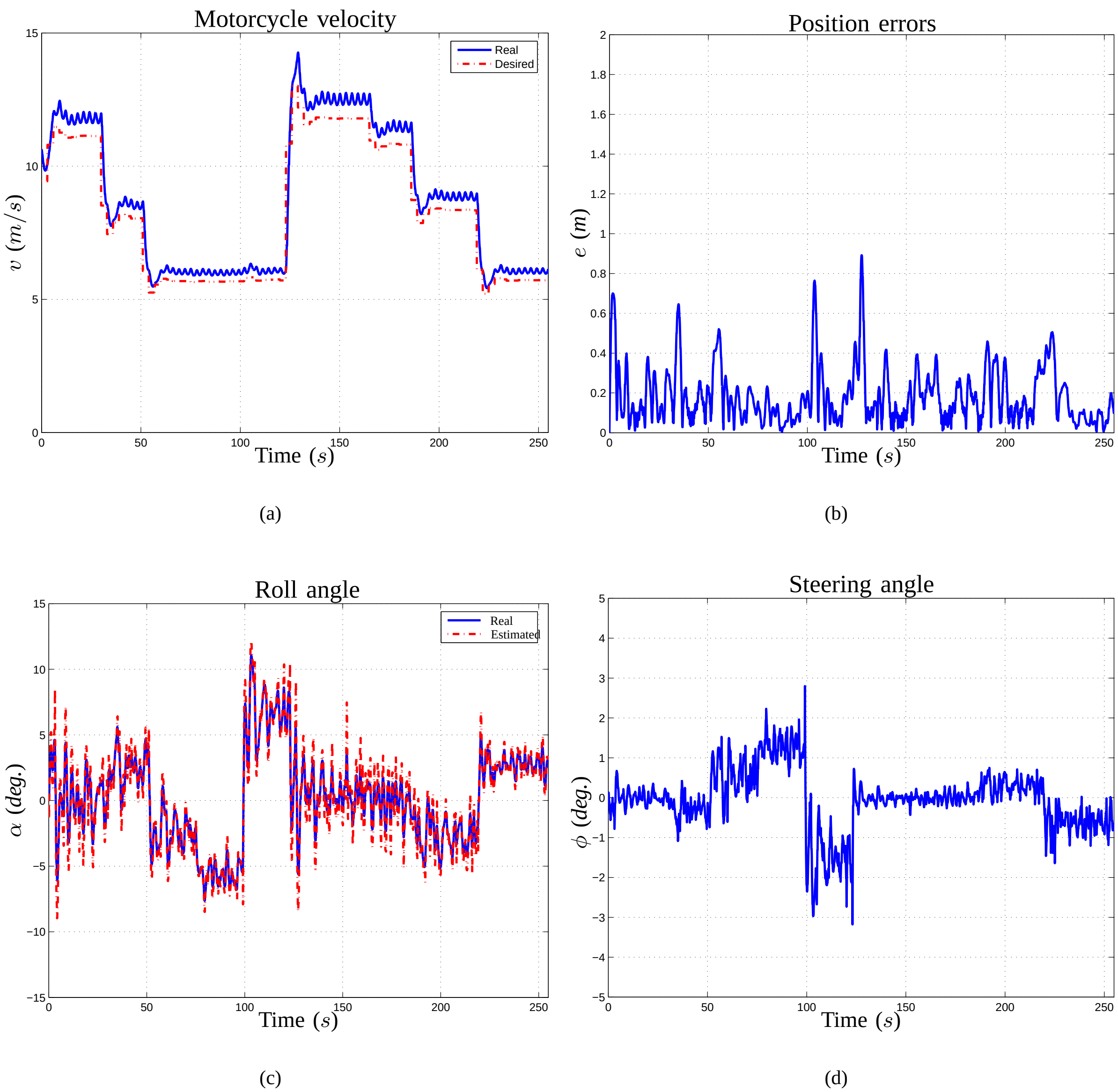


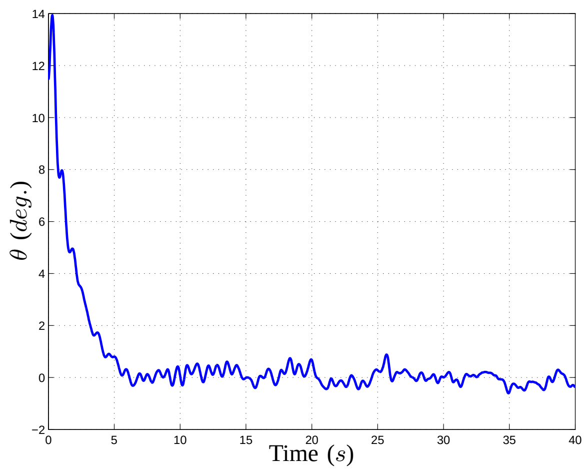
Fig. 6. Simulation results of the motorcycle trajectory tracking control with measurement noise. (a) velocity, (b) position error with center of the road, (c) roll angle θ, (d) controlled steering angle ϕ.

curvature. Smoothing the velocity profile is not considered in the simulation, and thus there could be a significant velocity change if the trajectory curvature changes suddenly. In the simulation, we add white noise with standard variations 0.02 m/s, 0.005 m/s^2, $0.3°$, and $0.6°$ for linear velocity and acceleration, roll angle, and yaw angle, respectively.

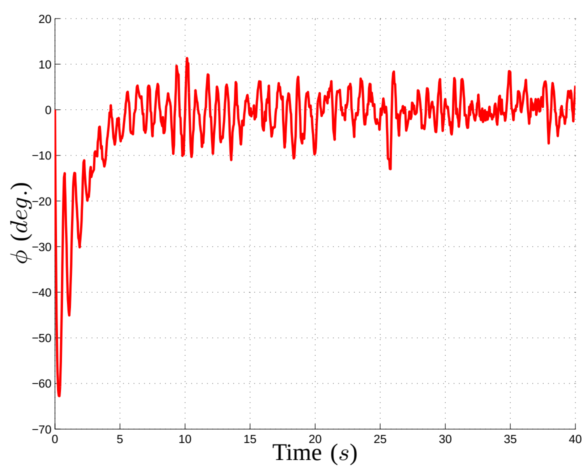
Assuming that the motorcycle position is known exactly (by GPS), we find that the integrated trajectory tracking controller works very well, and the position errors (defined as the difference with the desired trajectory) are within 1 m (Fig. 6(b)). Fig. 6(a) shows that the motorcycle velocity follows the desired values given by motion planning. Since the trajectory turning radii are not small (the smallest one is around 40 m), the controlled steering angle ϕ shown in Fig. 6(d) is within a $2°$ magnitude range. However, the roll angle θ shows a significant change during the motorcycle turns (Fig. 6(c)). The high frequency variations in the steering and roll angles come from added sensing noise and fast dynamics given in the motorcycle models. We believe that the mechanical damping of real system components will smooth signals in the experimental results more than those shown in Fig. 6.

The simulation results shown in Fig. 6 clearly demonstrate that the integrated vision/GPS-based motion planning and trajectory tracking control systems perform very well.

2) Balancing control with zero velocity: For motorcycle stabilization control at zero velocity, we have tested the control laws in the simulation. Fig. 7 shows the simulation results of the motorcycle roll angle and the controlled steering angle under an initial roll angle $\theta_0 = 11°$. The measurement noise for the roll angle θ and roll angle rate $\dot{\theta}$ are modeled as white noises with standard variation $0.6°$ and $0.6°/s$. From the simulation results, we can clearly see that the designed controller can stabilize the motorcycle from a relatively large disturbance when the vehicle is still.



(a)



(b)

Fig. 7. Simulation results of the motorcycle stabilization control under zero velocity and measurement noises. (a) roll angle θ, (b) controlled steering angle ϕ.

VII. Conclusions

In this paper, we present a mathematical modeling and a trajectory-following control system of an autonomous motorcycle system. We first develop a mathematical model for the autonomous motorcycle. Compared with the existing motorcycle dynamic models, the proposed model can capture the realistic motorcycle dynamics with a manageable complexity from the perspective of control system design. The trajectory following and self-balancing controller is developed based on a nonlinear control approach, and asymptotic tracking capability is guaranteed. The desired trajectory profile is generated using GPS/vision-based algorithms, which have been tested experimentally. The simulation results show that the proposed autonomous motorcycle navigation and control systems can work well on an arbitrary narrow track. The experimental implementation and validation of the proposed algorithms are undergoing, and we will report the results once they are available.

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