Dynamic Medial Axis Based Motion Planning in Sensor Networks

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Abstract

An important property in sensor networks is the monitoring of temporal changes of hazardous situations such as forest fires. Rescue groups need to be aware of dynamic changes that affect their rescue efforts. In this paper, we discuss an infrastructure for sensor networks that provides a good abstraction of geometric and topological features of a dynamically changing sensing environment. This infrastructure enables efficient path planning and navigation using localized algorithms. We propose a dynamic medial axis infrastructure that represents shapes and changes of shapes in a geometric space. We develop distributed algorithms for maintaining this infrastructure as changes occur. Dynamic medial axis allows rescue teams to find a short path to safety in a changing environment. We show that our dynamic medial axis algorithms have low message complexities and provide good approximations to the true medial axis.

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1 Overview

We propose a geometric abstraction of sensor networks that captures the changes of the network topologies and provides an efficient path planning scheme based on the medial axis [2]. Defined as the set of points with at least two closest neighbors on the boundaries of the shape, the medial axis is a skeleton of a region that preserves both of its geometric and topological features. Therefore it has been employed in areas such as robot motion planning and surface reconstruction [1], to explore properties of shapes. Recently there has been a proposal [3] of using medial axis as an abstraction of communication network of a sensor field. We show that the medial axis can be dynamically constructed and efficiently modified to reflect the changing network topology, and can be compactly represented by a graph of a size proportional to the number of geometric features. Then we explore an efficient motion planning algorithm that can reflect the temporal properties of the network topology.

Our scheme uses medial axis as a compact abstraction of the sensor field topology, as done in MAP. The difference between our work and MAP includes the capture of dynamic changes of networks. MAP works only in static network topologies and takes no consideration of topological changes. Our scheme is designed to handle topological changes of the sensor field and their effects on path planning.

Our construction algorithm of the initial medial axis is similar to MAP. First, the boundary of sensor field is discovered by selecting a set of sensors on the boundary of the field and connecting nearby sensors. The boundary sensors start the broadcasting of messages to initiate the construction. The messages are either re-broadcast by neighboring sensors or dropped by the sensors that receive multiple identical messages. A sensor that receives messages from more than one sources becomes a *medial axis node*, which is defined as a node with at least two closest sensor nodes on the boundary. A medial axis node confirms its routes to the boundary by sending messages to the nodes from which it received messages. The medial axis is composed of all the medial axis nodes, represented by the thicker dotted lines in Figure 1. The thinner dotted lines in Figure 1 represent paths from a medial axis node to the boundary of the sensor field.

As the network topology changes continuously, the medial axis is modified to capture the emer-

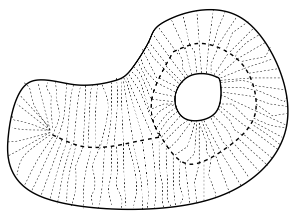

Figure 1: Medial axis, medial axis nodes, and their routes to the boundary

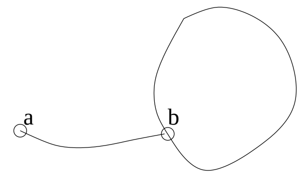

Figure 2: A medial axis graph

gence of new features, the change of the shape of existing features, the merging of multiple features, and the disappearance of features. Messages are initiated from the nodes on the changed boundary of topological features to the medial axis nodes, which then adjust the position of medial axis by transferring the duty of the medial axis node to some other sensors accordingly. All the adjustments to the medial axis are performed using only local knowledge with limited global information and are communicated through a small portion of sensor nodes.

The medial axis is represented by a medial axis graph (MAG); a graph of a size proportional to the number of geometric features. This MAG is compact and is known to every medial axis sensors. Figure 2 shows the MAG of Figure 1, which has two vertices, one edge, and one self-loop. The graph is dynamically updated along with medial axis adjustments. For each medial axis node u, we define a *chord* as the shortest path from u to one of its closest sensor nodes on the boundary. The sensors on the chord keep record of its distances to the corresponding medial axis node and boundary node. The medial axis nodes together with the chords provide a backbone communication and route system.

The goal of our path planning scheme is to find an approximate shortest path that avoids the changing obstacle regions. Our scheme supports localized routing with only the knowledge of the source and destination locations. It runs in two phases, first a global planning phase on the MAG, and then the execution of routing on each medial axis node. In the first phase an approximate shortest safe path from source to destination is found. The actual routing is taken place as each sensor node guides the mobile agent on a safe and short path. The maintenance of the medial axis infrastructure and the path planning and execution are all performed in a localized manner, therefore our schemes are scalable.

Our contributions. First, our dynamic medial axis based infrastructure captures the continuous geometric and topological changes of a sensor field. Second, the medial axis can be represented by a graph of a size proportional to the complexity of geometric and topological features. Third, the construction and dynamic adjustment of the medial axis is lightweight. Fourth, our medial axis based path planning algorithm is efficient, localized, and scalable.

2 Properties of Dynamic Medial Axis

Let M denote a bounded open set in $\mathbb{R}^2$ and ∂M the boundary curve of M.

Property 1: For any spatial variation of the boundary curve of M, there is a temporal representation of the variation. For any point p of ∂M, at time t_i, the location is r_i, with the *change* δr being $r_i - r_{i-1}$.

Lemma 2.1. *No two chords intersect except at the medial axis points.*

Proof. Suppose two chords b and c intersect at point z that is not on medial axis. For any point that is not on medial axis, it has only one closest point on ∂M. But z is on two chords b and c, therefore z has two closest points on ∂M, a contradiction. $\square$

Lemma 2.2. *A change to the boundary results in a change to the medial axis that remains a continuous line segment.*

Proof. To prove this, we show that for any changed medial axis point a'_1, we can find another changed medial axis point a'_2 within distance ϵ_a for any $\epsilon_a > 0$. Suppose two points b_1 and b_2 on ∂M. The distance $d(b_1, b_2) = \epsilon$. Suppose b_1 moves distance δ_1 to b'_1, and b_2 moves distance δ_2 to b'_2. Let δ be the difference between δ_1 and δ_2 ($\delta = d(\delta_1, \delta_2)$). Let $d(b'_1, b'_2) = \epsilon_a$, then $\epsilon_a \geq \sqrt{\epsilon^2 + \delta^2}$.

Let b'_1 be the tangent point of maximal disk centered at a'_1. Suppose, for any point b' on the new boundary between b'_1 and b'_2, the corresponding medial axis point a' is not within distance ϵ_a to a'_1.

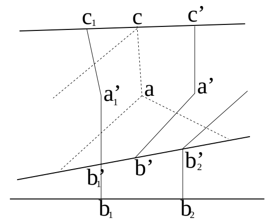

Figure 3: Illustration for the proof of Lemma 2.2

Suppose point c_1 on ∂M is another tangent point of maximal disk centered at a_1', and point c' on ∂M is another tangent point of maximal disk centered at a'. Then for any point c on ∂M, inbetween c_1 and c', let a be the center of the maximal disk tangent at c. If a is not inbetween a_1' and a', then chord ac intersects chord $a_1'c_1$ or $a'c'$. This is a contradiction to Lemma 2.1. Therefore a must be inbetween a_1' and a'. Then since the other tangent point of maximal disk centered at a must be outside of b_1' and b_2', the chord through the tangent point and a causes an intersection with $a_1'b_1'$ or $a'b'$, a contradiction to Lemma 2.1. Therefore, we conclude that there must be some medial axis point within distance ϵ_a from a_1'. This is illustrated in Figure 3. $\square$

Lemma 2.2 provides the proof for the following theorem.

Theorem 2.3. *The property of the (static) medial axis holds true in the dynamic medial axis.*

References

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