Integrating a Spatial Reasoner with a Resolution Theorem-Prover

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Recent interest in spatial/diagrammatic reasoning

- Special issue in *Computational Intelligence*, 1993.
- *Diagrammatic Reasoning*, Glasgow et al. (eds.). AAAI Press, 1995.
- AAAI Symposia on Reasoning with Diagrammatic Representations (Fall 1992, Spring 1997)

Various models of spatial reasoning

- Kosslyn: image construction, transformation, and inspection as a model of cognition
- Funt; Joskowicz: simulations of the physical world
- Forbus: metric diagram/place vocabulary for qualitative physics
- Glasgow: array theory

Contrast with attempts to capture semantics of spatial relations in logic (Mukerjee; Kaufmann; Davis)

Primary benefit of Spatial Reasoning:

- visualization = model-building
- fast lookup procedures = compiled inference

Disadvantages:

- restricted to spatial domains
- not good for other types of inferences

Question:

- can we integrate specialized reasoners?
- e.g. use resolution to handle logical/non-spatial
- use each component where most appropriate

Algorithm

Reasoning sub-systems:

- **Logical**: resolution refutation
- **Spatial**: ISR (Ioerger, 1995)

  Given some assertions, generate a plausible image, or return INCONSISTENT.

Overview

1. Pick a random image.
2. *Specialize* domain theory w.r.t. image.
3. Run resolution till derive $\Box$.
4. Analyze proof.
5. Put key spatial literals in *Covered List*.
6. Generate new image that avoids condition.
7. If Step 6 succeeds, goto Step 2. Else halt.

Specialization of Domain Theory (given an image)

- **if a spatial literal is true, drop clause**

  { $\neg$*On(book,table)* $\vee$ *Owns(pat,book)* }

- **if a spatial literal is false, annotate clause**

  { *Owns(pat,book)* } / { $\neg$*On(book,table)* }

Analyzing Proof

- Collect annotated spatial literals in clauses used in derivation of $\Box$.

- If conditions are ever met again, can re-derive.

- Place negations of literals in Covered List (DNF):

  $Cov = (\neg\alpha_1 \wedge \neg\alpha_2) \vee (\neg\beta_1 \wedge \neg\beta_2)...$

- Pick at least 1 lit. from each term, negate it, and force true in all subsequent images: e.g. $\alpha_1 \wedge \beta_1$

- Covers whole classes of images at a time.

- Intelligent search for consistent models.

Proof

What have we done to the completeness of resolution?

- TH (with negated query) is not nec. unsatisfiable
  e.g. { left(A,B) $\wedge$ right(A,B) }

- must consider interpretation w.r.t. SR

- if TH is unsat. "in context of SR", will alg halt?

SR is subset of restricted models that are legal images.

- P projects models of theory to just spatial preds.

- if $\mathcal{M}$ is the set of all models, then $SR \subset P(\mathcal{M})$.

- "unsatisfiable": $P(M(TH)) \cap SR = \emptyset$

Lemma: *If TH is "unsatisfiable", then the specialization of the theory TH for any image I, $Spec(TH, I)$, will entail $\Box$.*

Proof:

- If $P(M(TH)) \cap SR = \emptyset$, and $I \in SR$,
  then $P(M(TH)) \cap I = \emptyset$.

- This implies $M(TH) \cap P^{-1}(I) = \emptyset$.

- Finally, $M(Spec(TH, I)) = M(TH) \cap P^{-1}(I)$.

Theorem: *If TH is "unsatisfiable", then Alg will halt.*

Proof:

- With each iteration, we generate a new image not covered by the Covered List (via literal selection).

- When $\Box$ is derived, the annotated spatial literals are guaranteed to cover this image (and more).

- So adding to Covered List forces it to grow.

- Since the space of possible images is finite, it will eventually be exhaustively covered.

Relation to method of Myers and Konolige (1995)

- involves augmenting logical inferences by incremental model-building
- as spatial facts are derived, reflect into image
- requires use of "structural uncertainty"
- quite different from ISR's dynamic manipulation
  - M & K: must decide how to encode uncertainty in data structures
  - ISR: must design image manip. operators

Relation to hybrid-reasoning/constaint-logic program.

- could treat SR as extensions of unification (e.g. theory resolution; Stickel, 1985)
- could attach a constraint-solver to determine when derivations of $\Box$ in multiple contexts covers all cases (Burckert, 1994)

Limitations

- *universal quantifiers*:

  domain enumeration; scalability?

- *existential quantifiers*:

  disallowed; must know all objects in image

Applications

- reasoning about reactions in organic chemistry
  - *logical:* target/attack groups, solvent effects, valence, partial charges, resonance
  - *geometric:* steric effects, chirality
- GIS: add logical reasoning to spatial data (maps)
- integrating fuzzy-logic models of systems with FOL domain theories
  - *engine model:* air/gas mixture, temperature, pressure, firing rate, gear ratios, power
  - *parts data:* cost, reliability, toxicity, availability