## Just for Fun...Related Info on Robot Motion Planning and Configuration Space

Although this problem is simple to conceptualize in 2D, it is applicable to much more complex tasks, including planning of motions for articulated robots. A typical robot arm, as might be seen in a manufacturing plant, has several joints that can be rotated, such as an elbow wrist, and fingers on the gripper. The robot arm can be put into many positions by manipulating these rotation angles. The challenge is to find a sequence of moves that will maneuver the arm in the workspace into a goal position (e.g. to grip a part) without crashing into anything (e.g. fixtures such as a conveyor belt, roof, other machines). Such an environment in naturally 3D. However, the problem is usually modeled more abstractly in higher dimensions corresponding to all the degrees of freedom.

some of these images are taken from Wikipedia.

For example, a typical robot with at least 6 articulations (including being able to spin around the base) has 6 degrees of freedom, meaning that it can place the tip of the arm at any point (within range) in any orientation (think of a drill or paint nozzle). Motion planning is conceived as finding a path in the 6dimensional space (called "C-space" or configuration space), where each dimension represents a rotation angle (rather than physical $X, Y$, or $Z$ ). Based on the locations of obstacles in the workspace, regions of this 6-dimensional space are occluded, representing combinations of settings that would cause a collision between the robot and an object. Although it hard to visualize, these occluded regions
in 6D space are very analogous to the obstacles in 2D space, except the "images" of objects in C-space are usually curved surfaces rather than polygonal objects.

some of these images are taken from http://www.cs.cmu.edu/~motionplanning/lecture/Chap3-ConfigSpace_howie.pdf

As another example, consider navigating city streets in an 18-wheeler truck. It is not just a point - it has an articulation angle. What is even more complicated about this example is that you cannot arbitrarily set (rotate) this angle. Instead, getting the truck around obstacles and down narrow streets might involve steering, moving, and backing up to get the truck in the right position to move forward. The constraints in this problem are called "non-holonomic", and it makes motion planning quite difficult. A related problem, but without the articulation of a truck, is parallel-parking a car.

## Way points

In the example of the ATM shown above, the way points illustrated are on a regular grid. However, there are many other approaches to covering an environment with way points. In an environment with polygonal objects (see example 3.31 in the textbook), the waypoints may be equated with vertices of objects. It is sufficient to find a path the travels from vertex to vertex, to get to the goal. Note that we typically model the robot as a "point" (of no physical size), so it can get arbitrarily close to an obstacle. However, if the robot has a "width" or you prefer to maintain a "clearance" from objects, one approach is just to "expand" the size of the obstacles by the desired buffer, and eliminate consideration of way points that get too close.


One of the limitations of the regular grid is that it is relatively easy to missing having a critical way point in a narrow channel. In complicated environments, you might have to increase the resolution of the grid to ensure that enough way points appear to allow representation of and access to intricately defined conformations. (such as between the M and the T in ATM) Another approach is to generate random way points, called Probabilistic Road Maps. This is being studied by Dr. Amato's group. The idea is to generate way points with random locations. The density of points can be chosen to correspond with the complexity of the environment. Then you have to "tesselate", or add edges between proximal waypoints to form a graph (similar to a Delauney triangulation). One can prove some nice theoretical properties about the ability to find paths in PRMs.


