# CPSC 420 – Homework 1
due: Wednesday, March 21, 2007

1. Determine whether the following sets of propositional formulas are valid, satisfiable (but not valid), or unsatisfiable by giving a truth table for each. Be sure to label each table with your conclusion (valid, etc.), and indicate which truth assignments are satisfying.

    a. $\{P \rightarrow (Q \rightarrow P)\}$
    b. $\{(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))\}$
    c. $\{(P \wedge (P \rightarrow Q)) \rightarrow Q\}$
    d. $\{(P \wedge (P \rightarrow Q)) \rightarrow \neg Q\}$
    e. $\{((M \rightarrow X) \wedge \neg X) \rightarrow \neg M\}$
    f. $\{A \rightarrow C, B \rightarrow C, A \vee B, \neg C\}$
    g. $\{P \rightarrow Q, Q \rightarrow R, P \vee Q, \neg Q \vee \neg R\}$
    h. $\{(A \oplus B \oplus C), \neg(A \oplus B), \neg(A \oplus C), \neg(B \oplus C)\}$

2. Show that the proposition A is entailed by the following set of sentences by deriving a proof via Natural Deduction (i.e. using rules of inference). Recall that a proof (or derivation) contains a sequence of sentences labelled with the indexes of the sentences they were derived from, along with an indication of the inference rule used. (hint: think of various syntactic transformations you can apply to the initial sentences.)

    $\{D \wedge E \rightarrow A, F \wedge C \rightarrow D, \neg(F \wedge C \rightarrow B), C \vee B \rightarrow E\}$

3. You are the proprietor of Sammy's Sport Shop. You have just received a shipment of three boxes filled with tennis balls. One box contains only yellow tennis balls, one box contains only white tennis balls, and one contains both yellow and white tennis balls. You would like to stock the tennis balls in appropriate places on your shelves. Unfortunately, the boxes have been labelled incorrectly; the manufacturer tells you that you have exactly one box of each, but that each box is definitely labelled wrong.

incorrect:

| white | yellow | both |

observations:

(yellow)   (white)   (yellow)
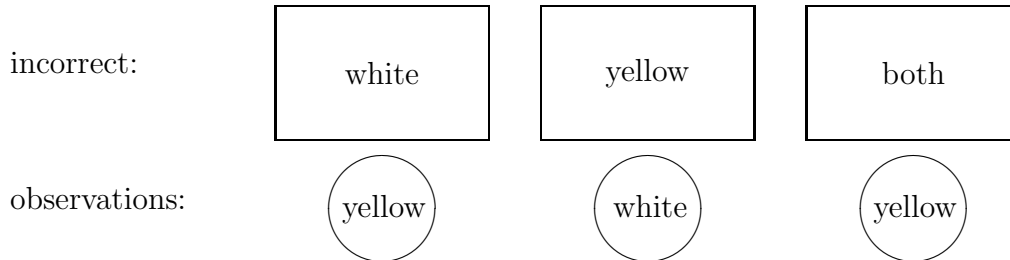
Given the initial (incorrect) labelling of the boxes above, and the three observations, use Propositional Logic to derive the correct labelling of the middle box. Begin by writing down a knowledge base you will need (such as what observing a white ball drawn from box 2 implies, that at most one box can contain yellow balls, etc.), along with the initial facts. Use propositional symbols in the following form: **O1Y** means a yellow ball was drawn (observed) from box 1, **L1W** means box 1 was initially labelled white, and **C1B** means box 1 actually contains both types of tennis balls.

a) Use Natural Deduction to prove that box 2 contains white tennis balls (i.e. generate the sentence **C2W**).

b) Show that box 2 must contain white balls via a Resolution Refutation proof (may require converting some sentences to CNF).