CPSC 625 – Homework 5

due: Mon, Oct 30, 2006

0. Prove the following equivalence using model theory for FOL:
$\neg\exists x\ \Gamma(x) \equiv \forall x\ \neg\Gamma(x)$, where $\Gamma(x)$ is a sentence containing the variable x.

1. Prove that Existential Introduction is a sound rule of inference in FOL: $\Gamma(a) \vdash \exists x\ \Gamma(x)$, where $\Gamma(a)$ represents some sentence that contains the constant term a, which is replaced by the (new) variable x throughout.

2. Consider the following model for a first-order logic theory containing the constant a, the function f, and predicates P and Q: universe is $\mathcal{U} = \{1, 2\}$; denotations are $a \mapsto 1$, $f(1) = 2$ and $f(2) = 1$; extensions are $P = \{\langle 2\rangle\}$, and $Q = \{\langle 1,1\rangle, \langle 1,2\rangle, \langle 2,2\rangle\}$. Determine the truth value of the following sentences in the model. Be sure to show all of your work.

a) $\forall X\ P(X) \to Q(f(X), a)$
b) $\exists X\ P(f(X)) \wedge Q(X, f(a))$
c) $\exists X\ \neg P(X) \wedge Q(X, a)$
d) $\forall X\ \exists Y\ P(X) \wedge Q(X, Y)$
e) $\neg\exists X\ \forall Y\ Q(f(Y), X)$

3. Show that the following first-order theory is satisfiable by giving a model that satisfies it. (In this problem, X and Y are variables, while a and b are constant terms.)

$$\{\forall X\, \forall Y\, P(X,Y) \to Q(Y), P(a, f(b)), \neg Q(a) \wedge \neg Q(b)\}$$

4. Determine whether or not the following pairs of predicates are are unifiable. If they are, give the most-general unifier and show the result of applying the substitution to each predicate. If they are not unifiable, indicate why. Variables are in capital letters; constants are lowercase.

a) $P(a, X, f(g(Y)))$ and $P(Z, f(Z), f(U))$
b) $Q(f(a), g(X))$ and $Q(Y, Y)$
c) $R(f(Y), Y, X)$ and $R(X, f(a), f(V))$
d) $P(a, Y, f(X))$ and $P(X, f(b), f(b))$
e) $Q(g(f(a)), g(X), Z)$ and $Q(Y, Y, f(X))$
f) $P(a, X, g(f(f(a)), X))$ and $P(Z, f(Z), g(Y), f(Z)))$
g) $Q(f(a, a), Y, Z)$ and $Q(X, f(Z, Z), Y)$

5. Using first-order rules of inference, prove that "there exists a vegetarian" from the following pieces of knowledge: anyone who does not eat meat is a vegetarian, tomatoes are not meat, carrots are not meat, and there is someone who eats only tomatoes and carrots. The initial sentences (premises) are translated into first-order logic for you below. The goal is to generate: $\exists X\, vegetarian(X)$. Be sure to explicitly label each new sentence with the one(s) it was derived from, along with the inference rule and any substitution used.

$\forall P\, (\forall X\, eat(P, X) \to \neg meat(X)) \to vegetarian(P)$
$\forall X\, tomato(X) \to \neg meat(X)$
$\forall X\, carrot(X) \to \neg meat(X)$
$\exists P\, \forall X\, eat(P, X) \to (tomato(X) \vee carrot(X))$