1. Given the following, can you prove that unicorns are magical? “If the unicorn in mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.”

2. If the congress refuses to enact new laws, then the strike will not be over unless it lasts more than one year and the CEO of the company resigns. If congress enacts new laws or the strike is not over then the strike last more than one year. Are these statements contradictory? If so, prove they are unsatisfiable. If not, give a satisfying truth assignment (and show it satisfies the sentences).

3. You are the proprietor of Sammy’s Sport Shop. You have just received a shipment of three boxes filled with tennis balls. One box contains only yellow tennis balls, one box contains only white tennis balls, and one contains both yellow and white tennis balls. You would like to stock the tennis balls in appropriate places on your shelves. Unfortunately, the boxes have been labelled incorrectly; the manufacturer tells you that you have exactly one box of each, but that each box is definitely labelled wrong.

   incorrect:
   
   observations:

   Given the initial (incorrect) labelling of the boxes above, and the three observations, use Propositional Logic to derive the correct labelling of the middle box. Begin by writing down a knowledge base you will need (such as what observing a white ball drawn from box 2 implies, that at most one box can contain yellow balls, etc.), along with the initial facts. Use propositional symbols in the following form: \(O_{1Y}\) means a yellow ball was drawn (observed) from box 1, \(L_{1W}\) means box 1 was initially labelled white, and \(C_{1B}\) means box 1 actually contains both types of tennis balls.

   a) Use rules of inference to prove that box 2 contains white tennis balls (i.e. generate the sentence \(C_{2W}\)).

   b) Show that box 2 must contain white balls via a resolution refutation proof (may require converting some sentences to CNF).