Investigation of Density and Flow Relationships in Congested Traffic Using Videogrammetric Data

Thomas R. Ioerger[1], John H. Meeks, and Paul Nelson

Department of Computer Science, Texas A&M University

301 H.R. Bright Building, College Station, TX 77843-3112

phone: (979)845-5534, fax: (979)847-8578

ioerger@cs.tamu.edu, meeks@tamu.edu, pnelson@cs.tamu.edu

manuscript length: 3920 words + 10 figures

[1]Corresponding author

Abstract

Traffic congestion is a major problem on our nation's highways. Studies and modeling of congestion can have important implications for efficiency, safety, and planning of transportation systems. A variety of theoretical models have been proposed to explain the behavior of congested traffic, including the recent interesting postulation of "synchronized flow" states, and treatment of these as phases (in the statistical mechanics sense). However, empirical methods for identifying and characterizing such patterns of congestion have been hampered by lack of objective and quantitative characterizations of these patterns. In this paper, we present an empirical study of the patterns of traffic in a dataset from a congested, 5-lane basic section of freeway. This data was collected using videogrammetry, which provides additional information (on the spatial variation), in comparison to other sensor technologies. Using this data, we performed a macro-analysis of relationships among flow, density, and velocity, and we found a linear (decreasing) relationship between velocity and density, with the attendant quadratic of flow on density. Through a micro-analysis, we observed that depressions in velocity under congested conditions appear to propagate backward as "constrictions" (waves of reduced flow), and we designed a signal-detection algorithm for recognizing these events in a time series based on convolutions of the density gradient.

Keywords: traffic congestion, synchronized flow, fundamental diagram, videogrammetric data

INTRODUCTION

Traffic congestion is a complex but important phenomenon that needs to be better understood for numerous purposes, ranging from safety management on our nation's highways, to planning and development of new highway systems to meet ever increasing transportation needs. Over the years, there have been a variety of theoretical approaches proposed for modeling congested traffic flow, including those based on queuing theory [1], kinematic wave theory [2], kinetic theory [3], higher-order models [4], cellular automata [5], and car-following models [6], among others. These models, particularly the kinematic-wave theory, have each contributed a unique perspective to our understanding of how high vehicular density develops along stretches of road, how this ultimately leads to a reduction in overall vehicle velocities, and how this situation evolves over time.

Yet it is important to tie such theories to empirical studies and analysis of traffic data. Many such analyses have been made, based on data from the point detectors that are both pervasive and likely to be the basic source of data for operational applications. But if the objective is to achieve a high level of understanding of actual traffic behavior, it seems appropriate to augment such studies with those using some of the relatively limited amount of higher-quality data that is also available. For example, both videogrammetry [7] and RF-transceiver tags can be used to track individual vehicles as they move through a road section, including lane changes. Given this new data at a more fine-grained level, the question we ask is whether it can be used to better detect various phases of traffic congestion, and to quantify, measure, and validate some of the predictions made by the theoretical models.

Of note are recent studies that have proposed the existence of several distinct phases of congestion, collectively called "synchronized flow," that lie between free flow and stop-and-go traffic [8]. Further studies have demonstrated that such phases can be recognized in certain datasets, and efforts have been initiated to characterize their statistical properties [9]. These studies suggest that there is some structure in the traffic patterns at the point at which the fundamental relationship among density, velocity, and flow breaks down.

In this paper, we demonstrate possible approaches to quantitatively identifying patterns of traffic congestion, using videogrammetric data from a section of I-405 in Los Angeles [10]. This data records the lane, position, and velocity of each vehicle over multiple 1-second time frames, as it moves throughout the section. We provide both a macro-analysis of the fundamental density-flow relationship, and a micro-analysis of vehicular behavior around constrictions that helps explain how they develop and evolve. Through these studies, we show that:

1. while the flow-density curve does depart from a linear relationship at higher densities, the overall trend can be modeled as a quadratic function due to the apparent linear dependence of velocity on density that develops; and

2. although many small fluctuations in the density can be observed, on a small scale, from "platoons" of vehicles moving through, the fact that constrictions tend to propagate backward can be exploited to detect them by a time-series analysis of the (spatial) derivative of the density (local gradient of vehicle counts).

Remarkably, the first of the observations tends to confirm, at least on a statistical basis, the venerable traffic stream model originally suggested by Greenshields [11].

BACKGROUND

Simple intuition suggests that traffic congestion will occur in locations and at times where there is some type of bottleneck, such as an on-ramp, lane drop, or incident, and that this will lead to speed reduction and a spillback of congestion. The kinematic-wave theory agrees well qualitatively with these expectations (e.g. [12]), even with relatively simple traffic stream models (fundamental diagrams). Traffic congestion is not just restricted to physical locations where constrictions occur, but also occurs spontaneously due to slower-moving vehicles (well below the mean of the desired speed distribution). Clearly, this dynamic form of congestion is sensitive to vehicle density [13]. At lower densities, slow vehicles can easily be bypassed via lane changes. However, at higher densities, there is less opportunity to change lanes, and spillback can occur, causing large numbers of vehicles to slow down.

This behavior of congested traffic flow, where there is a reduction in velocity at higher densities, can easily be observed in a plot of the "fundamental diagram," which shows flow as a function of density (see Figure 1). In general, it is typically found that, at lower densities, flow is a linear function of density, producing a more-or-less straight line with a slope equal to the mean velocity. Thus, the more vehicles, the more flow, which is the assumption nominally. However, it has been well-documented that this linear relationships breaks down at higher densities [14]. If density/flow data are plotted in the high-density regime, they tend to form a cluster underneath the extrapolation of the flow line at low densities. Generally, this traffic is moving at lower velocities, and the scatter can range all the way down to the horizontal axis, representing a speed of 0 mph and hence stop-and-go traffic (jams).

Kerner and Rehborn [8] have used the term "synchronized flow" to distinguish this pattern of traffic that lies intermediate between free flow and stop-and-go states. The general concept of synchronized flow is that there is a strong tendency to equality, across adjacent lanes, in the velocities of nearby vehicles. Synchronized flow has been further divided into at least three sub-types: 1) where both the flow and velocity remain nearly constant over a long interval of time, 2) where the average velocity remains fairly constant, but the flow (and hence density) varies considerably, and 3) the average speed and velocity change dramatically over a short period of time. Various properties of synchronized flow have been investigated [15]. For example, whereas in free flow there appears to be two peaks in the time headway distribution, roughly at 0.8 sec and 1.8 sec, mainly the 1.8 sec peak is observed in synchronized flow [9]. Also, synchronized states reveal themselves in a break-down of the cross-correlation between flow and density, indicating less regularity in the traffic flow.

Synchronized flow has begun to be viewed as a phase, in the statistical mechanics sense [16]. This is because when traffic drifts into synchronized flow, it tends to persist in this state for some time. Flow becomes less a function of density, and instead becomes more dependent on its recent history (a kind of hysteresis). In fact, free flow at these higher densitities can be thought of as a meta-stable state, since it usually shifts to synchronized flow eventually. What causes this transition between free flow and synchronized flow is unknown, and the subject of some controversy [17]. Furthermore, it seems unclear how (if at all possible) to characterize the relationships among flow, density, and velocity for traffic in the synchronized flow phase. In the next sections, we explore empirical answers to these questions by examining a representative dataset.

THE MULHOLLAND TRAFFIC DATASET

In this section, we describe the dataset used in our empirical analyses of traffic congestion. We used the Mulholland dataset from the TFHRC website [10], which covers a 1341 ft section of road on the San Diego Freeway (I-405) in Los Angeles near the Mulholland Drive overpass. This is a straight, 5-lane section of highway with no exits, on-ramps, or intersections. The data was collected using a video camera, and was transformed into records describing vehicle lanes, positions, velocities, etc. [7]. A frame was recorded once each second, and each frame contains on the order of 30-80 vehicles. Significantly, identities of vehicles were assigned and tracked from frame to frame. This data allows more interesting analyses than traditional induction-loop detectors, because the history of a vehicle can be observed over time as it passes through the section.

The data was collected in June, 1983 and spans approximately one hour (3600 frames). During this time, traffic volume is fairly heavy, as this is a widely-used artery (average density is 39.1 vpml). Figure 2 shows the fluctuations in the density and the velocity over the whole observation period. As can be seen, there are several instances where significant slow-downs occur, generally in concert with peaks in the density, though complete stoppages are rare and brief. (In the original description of this dataset, it was noted that a vehicle became disabled in the right-hand lane at around 22 minutes into the observation period, and another vehicle stopped on the left-hand shoulder at around 38 minutes.) A useful representation of these constrictions is to plot individual vehicle positions in a given lane as a function of time. As an example, a plot of lane 5 (the left-hand lane, fastest) is shown in Figures 3a and 3b, showing a detailed picture of the traffic pattern for the constrictions that occur around frames 1300 and 2900, respectively.

MACROSCOPIC ANALYSIS: VELOCITY AS A FUNCTION OF DENSITY

As an analysis of the macroscopic behavior of the traffic congestion in this dataset, we begin by plotting the data on the fundamental diagram (see Figure 4). For the data points in this graph, the section was divided into 4 335-ft subsections, and the density ρ within a subsection was calculated as the total number of vehicles over all 5 lanes divided by the length L of the subsection ($L = 335$ ft),

$$\rho_{s,t} = \frac{1}{5 \cdot L} \sum_{l=1}^{5} n_{s,l,t},$$

where $n_{s,l,t}$ represents the number of vehicles in subsection s in lane l at time t. Flow q is calculated as the sum of the velocities, divided by the length of the subsection:

$$q_{s,t} = \frac{1}{5 \cdot L} \sum_{l=1}^{5} \sum_{i=1}^{n_{s,l,t}} v_{i,t},$$

where $v_{i,t}$ is the velocity of vehicle i in subsection s in lane l at time t. If one takes $v = v_{s,t}$ as the space mean speed corresponding to the vehicles in the subject subsection at time t, then this is the definition of flow mandated by the fundamental identity $q = \rho v$, in the manner of Edie [18]. The classic pattern is easily observed in this plot, where there is a linear relationship between flow and density for lower densities, but this relationship breaks down at higher densities. We aim to develop a better understanding of what relationships exist in this region of scatter, presumably corresponding to synchronized flow.

Because of the nature of the flow-density diagram, it can be inferred that vehicle velocities have decreased in this high-density regime. In fact, while there is significant scatter on this plot, a decreasing trend is observable. Hence we hypothesized that velocity might be a function of density. Figure 5 shows a plot of velocity with respect to density for the Mulholland dataset. From this plot it is abundantly clear that there is a functional relationship in the *trend* of velocity on density, and that the relationship is almost perfectly linear (with a negative slope). Many individual data points (representing specific subsections at specific times) vary from the precise linear relationship. Yet, in aggregate, the trend is clear. At low densities, vehicles are able to travel at whatever speed they desire, and extrapolation to $\rho = 0$ gives an intersect of $v_0 \sim 60$ mph, an estimate of the mean desired speed of drivers. As density climbs, velocity drops off. By the least-squares method, the best-fit line describing this data is given by the following equation (notice the negative coefficient on ρ):

$$v = v_0 + c\rho = 63.8 - 0.545 * \rho.$$

The correlations coefficient for this line is $R^2 = 0.573$.

Such a relationship between density and velocity has been hypothesized decades ago [11], and is well-known to lead to a quadratic dependence of flow on density,

$$q = q(\rho) = v_0\rho + c\rho^2.$$

Figure 6 superimposes this curve on the scatter plot from Figure 4. This curve is fairly consistent with the original linear relationships for lower densities, but also fits through the scattered points where the linear relationship breaks down. Hence we conclude that, in this dataset, velocity is a function of density, and this leads to a quadratic model of flow, as a function of density.

There is one final piece of evidence that supports this view. If we plot the flow over the whole observation period (see Figure 7; for this plot the flows were obtained by the methodology descibed in more detail in the following section), we observe that it is relatively constant spatially (despite minor fluctuations). In comparison to Figure 2, little perturbation in the flow is associated with the constrictions occurring at times such as frame 1300, 1600, or 2900. Thus, over the entire period of observation flow remains roughly invariant, forcing the inverse relationship between density and velocity.

MICROSCOPIC ANALYSIS: VELOCITY AS A FUNCTION OF DENSITY GRADIENT

At a microscopic level, the question arises as to what causes the onset of synchronized flow. In this section, we look in closer detail at the traffic patterns that occur in the Mulholland dataset to find an explanation for the link between velocity and density. Further, we seek a quantitative procedure for automatically recognizing this type of congestion in videogrammetric data, which has not been made clear in previous works.

From theoretical studies in kinetics, it has been proposed that drivers might decrease their velocities in *anticipation* of increased density in front of them [19]. Mathematically, this arises as a dependence on the (spatial) derivative of the density, a corrective term which arises specifically in the high-density regime. This anticipatory response seems very natural from an intuitive perspective. However, such quantities as derivatives of density are difficult to estimate robustly at a micro-level scale.

To begin to examine this phenomenon, we took an approach in which we analyzed the local flow, velocity, and density at the mid-point of the 1341-ft section of road. Velocity and density at a given time were simply estimated by looking at the vehicles in a 400-ft box downstream from the midpoint, velocity being averaged over the vehicles present in that box, and density being the number of vehicles in the box divided by 400/5280 mi. The flow, since it is not directly provided as with induction-loop sensors, was calculated in a more sophisticated way. At any given point in time, vehicles that are downstream of the midpoint, but were upstream of the midpoint within the last 5 seconds are counted as part of the flow. This crucially relies on the fact that vehicles are assigned identities and are tracked from frame to frame, since their positions must be correlated between frames. Using this approach, the estimated flow values track almost exactly with the product of the density and velocity estimates (data not shown).

Plots of the local density and velocity for lane 5 are shown in Figure 8 for frames 1000-1700, covering the constrictions at 1300 and 1600. These values are smoothed over windows of (the preceding) 10 frames (seconds) to remove noise. Prior to frame 1300, it is clear that density varies quite a bit on this small scale, but the average velocity remains fairly stable and high. However, when congestion occurs, there is a significant drop in the velocities to the range of $\sim$10 mph, and corresponding increases in density that are large in magnitude (up to around 100 vpml). A similar phenomenon is observed at around frame 1600.

Interestingly, after each constriction (throughout the whole observation period), the velocity stays suppressed for a considerable interval of time, for example, around 30 mph in frames 1400-1600 after the constriction at 1300. In general, the velocity recovers slowly, although the densities return to fluctuating in a lower range as before. It is these periods of time where the velocities are depressed that produce the scattered region in the fundamental diagram. One could propose that the most significant constrictions are merely extremes of a common underlying behavior (e.g. fluctuations in density). However, as the other parts of the graph show, significant fluctuations in density do not necessarily cause major fluctuations in velocity. Furthermore, when the density subsides, the velocity does not immediately return to its prior level.

We suggest that the observed constrictions in traffic flow are responsible for *initiating* the transitions into these lower-velocity states, corresponding to synchronized flow. Hence, by looking for such constrictions, we can recognize the onset of congestion due to synchronized flow. We can identify such events in a traffic stream by taking advantage of the observation that constrictions in traffic flow have a tendency to propagate backwards in space (upstream). This phenomenon is consistent with kinetic theory [19], and has been observed in other datasets (for example, see [20]).

The backwards propagation can easily be seen in Figures 3a and 3b. It is exactly these constrictions that are correlated with the starting points of major variations in density and velocity. We propose that these constrictions are primarily responsible for sending velocities plummeting, and are the events that initiate synchronized flow.

While slightly higher in density than typical traffic flow, constrictions are otherwise difficult to distinguish from "platoons" of vehicles (high density clusters) moving through the section; both appear as peaks. Hence it is hard to tell when a local increase in density will produce synchronized flow. However, platoons move forward in time, whereas constrictions move backwards. Thus, we can construct a method for recognizing constrictions in the following way. First, we calculate the density on both sides of the midpoint of the section. By taking the difference, we get a local "gradient" in the density, at a fixed point in time. Clearly, both platoons and constrictions will show positive peaks in the gradient, since in both cases, there is a point in time when there are more vehicles downstream from the midpoint than upstream. However, in the case of platoons, we expect this peak to *follow a dip* in the gradient, since just beforehand, the platoon was in the upstream part of the section, causing the opposite imbalance. On the other hand, for constrictions, we expect the peak in gradient to build-up without any forewarning, due to the backwards motion of the constriction.

Based on these observations, we can construct a quantitative classifier for detecting the onset of synchronized flow. We take an approach based on using template convolutions, which is common in time-series analysis and signal detection. The convolution of two function $f(t)$ and $g(t)$ can be described formally as:

$$\mathcal{C}(f, g; u) = f(t) * g(t) = \int_{-\infty}^{\infty} f(t)g(u-t)\ dt.$$

In practice, the signal is detected by creating a discrete array of weights that follows the shape of the signal we are trying to detect, sliding this along the whole time series, and at each time point, calculating a local weighted sum. To capture the unique characteristics expected of the density patterns associated with backward-moving constrictions, we first convolve each window in the gradient time-series againt one-half cycle of a $sin(x)$ curve, which approximates a large pulse in the density gradient. Then we convolve the gradient time-series against a full cycle of the $-sin(x)$ curves, which approximates the oscillation due to forward-moving platoons (see Figure 9). We subtract this from the first convolution value to filter out pulses due to platoons, and square the resulting value get the energy of the signal (positive). In sum, the detector can be described as follows:

$$\sigma(u) = \{\mathcal{C}(g, sin(t; 0..\pi)) - \mathcal{C}(g, -sin(t; 0..2\pi))\}^2,$$

where $g(t)$ is the time-varying gradient signal.

An important parameter in computing this convolution is the window size W to use for the filter functions. We systematically tested a range of windows sizes between $W = 20$ sec and $W = 80$ sec, since this represents the range of durations of the peaks in the gradient for the various constrictions observed. The optimal value for the window size was found to be $W = 50$ sec. Figure 10 shows the convolution of the density gradient using this window size. This signal-detection method produces sharp peaks quite close to the beginning of each major down-turn in velocity. Lower values of W (e.g. 30 sec) generated spurious peaks in regions without recognizable constrictions, and larger values (e.g. 60 sec) produced broader, less-sharp peaks. The peaks obtained with $W = 50$ correspond well to the occurrences of constrictions in this dataset, and the subsequent onset of synchronized flow. This behavior instantiates the theoretical prediction that, at high densities, velocity may become sensitive to the derivative (gradient) of the density, perhaps due to drivers slowing down as they approach more congested regions.

CONCLUSION

In this paper, we have presented an empirical study of the patterns of traffic congestion in a dataset from a 5-lane basic section of freeway. This data was collected using videogrammetric technology, and thus provides additional information, in comparison to other detector methods, such as vehicle identities, positions, and velocities that can be tracked over time. Using this data, we performed a macro-analysis of relationships among flow, density, and velocity. We observed that, in this dataset, velocity tends to decrease linearly with increasing density on average, and this can be used to create a quadratic curve for flow as a function of density that fits the scatter of points on the fundamental diagram very well. On visual inspection of the traffic patterns, we noticed that significant down-turns in average velocity seem to correspond with "constrictions," or high-density clusters of vehicles that migrate backwards over time (as aggregates). Hence we defined local measures of density and its derivative (gradient) for a micro-scale analysis, and used these to design a numerical technique for automatically detecting these constrictions as signals in a time series using convolutions. We view these constrictions as explanations for the onset of synchronized flow, given that they act as triggering events that put the traffic into a lower-velocity state, from which it only slowly recovers.

Acknowledgements

This work was partially supported by a grant (to PN and TRI) from the Southwest Region University Transportation Center through the Texas Transportation Institute at Texas A&M University.

References

[1] G.G. Newell. *Applications of Queuing Theory.* London: Chapman and Hall, 2nd edition, 1982.

[2] M.J. Lighthill and G.B. Whitham. On kinematic waves II: A theory of traffic flow on long crowded roads. *Proceedings of the Royal Society of London*, A229:317–345, 1955.

[3] I. Prigogine and R. Herman. *Kinetic Theory of Vehicular Traffic.* New York: American Elsevier, 1971.

[4] R. Kühne and P. Michalopoulos. Continuum flow models. In N.H. Gartner, C. Messer, and A.K. Rathi, editors, *Traffic Flow Theory*, chapter 5. http://www.tfhrc.gov/its/tft/tft.htm, Accessed Jan. 15, 2000.

[5] K. Nagel and M. Shreckenberg. A cellular automaton model for freeway traffic. *Journal of Physics I, France*, 2:2221–2229, 1992.

[6] R.W. Rothery. Car following models. In N.H. Gartner, C. Messer, and A.K. Rathi, editors, *Traffic Flow Theory*, chapter 4. http://www.tfhrc.gov/its/tft/tft.htm, Accessed Jan. 15, 2000.

[7] L.A. Klein and M.R. Kelley. Detection technology for IVHS, Volume I: Final report. Technical Report FHWA-RD-95-100, US Department of Transportation, 1996.

[8] B.S. Kerner and H. Rehborn. Experimental properties of complexity in traffic flow. *Physical Review E*, 53:R4275–R4278, 1996.

[9] L. Neubert, L. Santen, A. Schadschneider, and M. Schreckenberg. Single-vehicle data of highway traffic: A statistical analysis. *Physical Review E*, 60(6):6480–6490, 1999.

[10] Turner-Fairbank Highway Research Center. http://www.tfhrc.gov/its/dataneed/opening.htm. Accessed May 1, 2000.

[11] B.D. Greenshields. A study of traffic capacity. In *Proceedings of the Highway Research Board 14*, pages 448–477. HWRB, National Research Council, Washington, D.C., 1934.

[12] G.F. Newell. A simplified theory of kinematic waves in highway traffic. *Transportation Research B*, 27B:289–303, 1993.

[13] K. Nagel and M. Paczuski. Emergent traffic jams. *Physical Review E*, 51(4):2909–2918, 1995.

[14] J.S. Drake, J.L. Schofer, and A.D. May. A statistical analysis of the Speed-Density hypothesis. In *Highway Research Record 154*, pages 53–87. HWRB, National Research Council, Washington, D.C., 1967.

[15] D. Helbing and M. Treiber. Gas-kinetic-based traffic model explaining observed hysteritic phase transition. *Physical Review Letters*, 81(14):3042–3045, 1998.

[16] B.S. Kerner and H. Rehborn. Experimental properties of phase transitions in traffic flow. *Physical Review Letters*, 79(20):4030–4033, 1997.

[17] C.F. Daganzo, M.J. Cassidy, and R.L. Bertini. Possible explanations of phase transitions in highway traffic. *Transportation Research B*, 33A:365–379, 1999.

[18] L. Edie. Discussion of traffic stream measurements and definitions. In *Proceedings of the Second International Symposium on the Theory of Traffic Flow*, pages 139–154. OECD, 1965.

[19] P. Nelson. Synchronized traffic flow from a modified Lighthill-Whitham model. *Physical Review E*, 61(6):6052–6055, 2000.

[20] B.S. Kerner and H. Rehborn. Experimental features and characteristics of traffic jams. *Physical Review E*, 53(2):R1297–R1300, 1996.

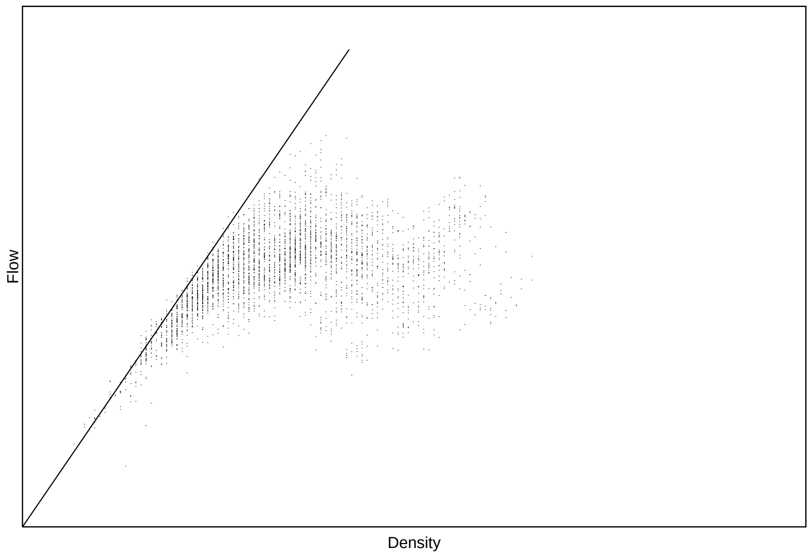

Figure 1: Fundamental Diagram.

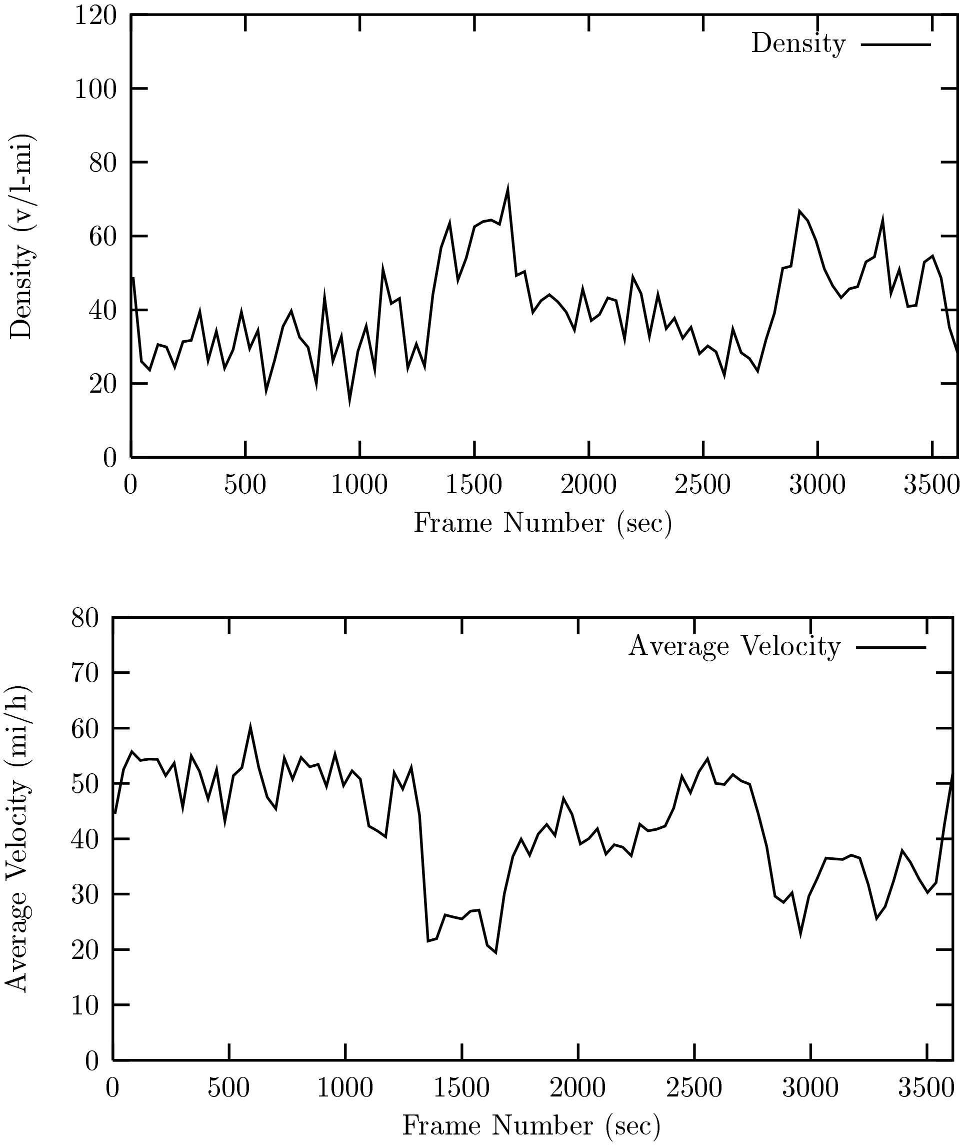

Figure 2: Density and velocity over the whole 1-hour time period, average over all 5 lanes in the Mulholland dataset.

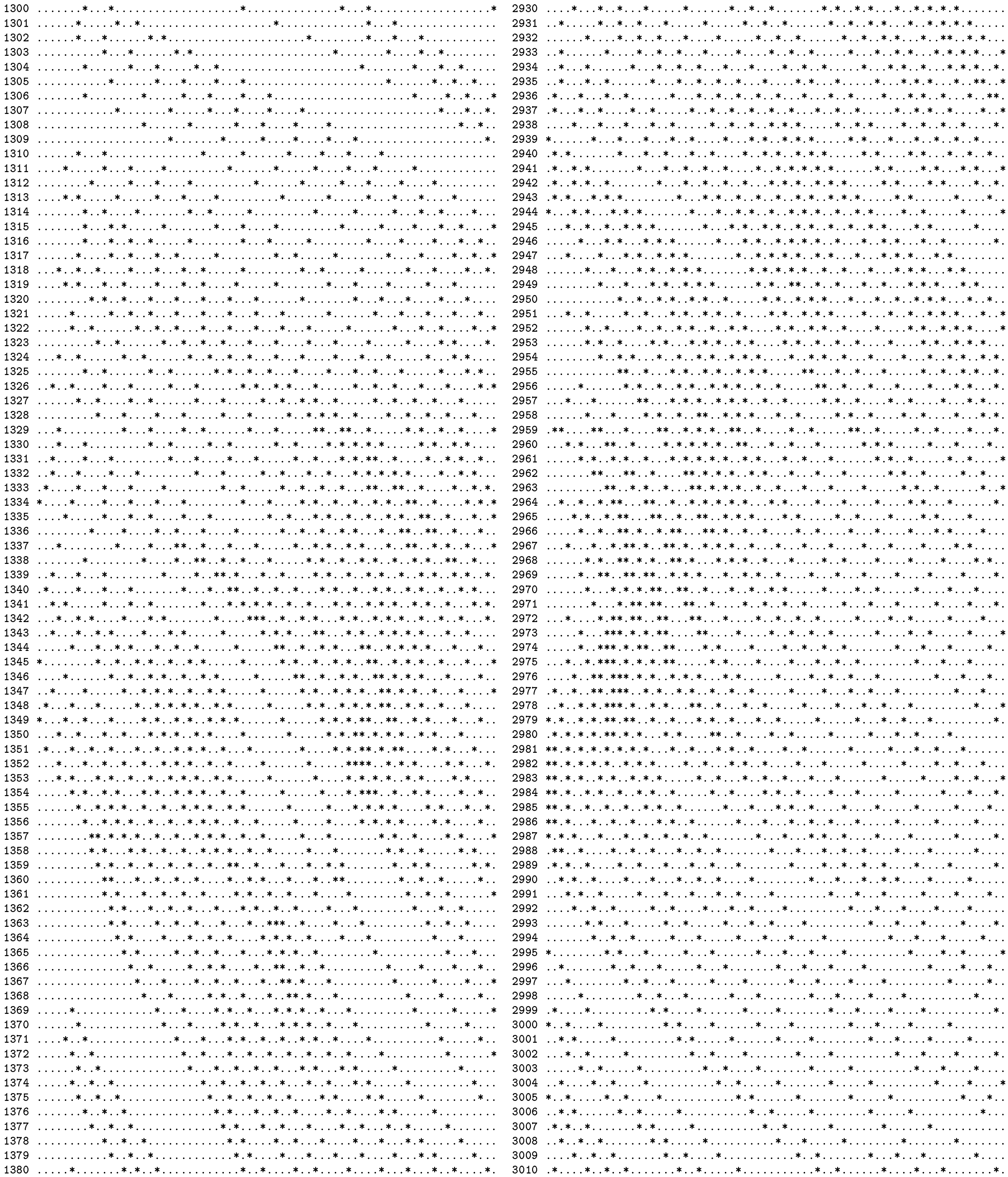

Figure 3: Visual representation of traffic in lane 5: a) frames 1300-1380, and b) frames 2930-3010. Each row shows the position of vehicles in lane 5 along the 1341-ft section. As time increases (downward), the vehicles can be seen to move forward (to the right). Curved trajectories indicate accelerating or decelerating vehicles. Both of these regions illustrate congestion due to constrictions.

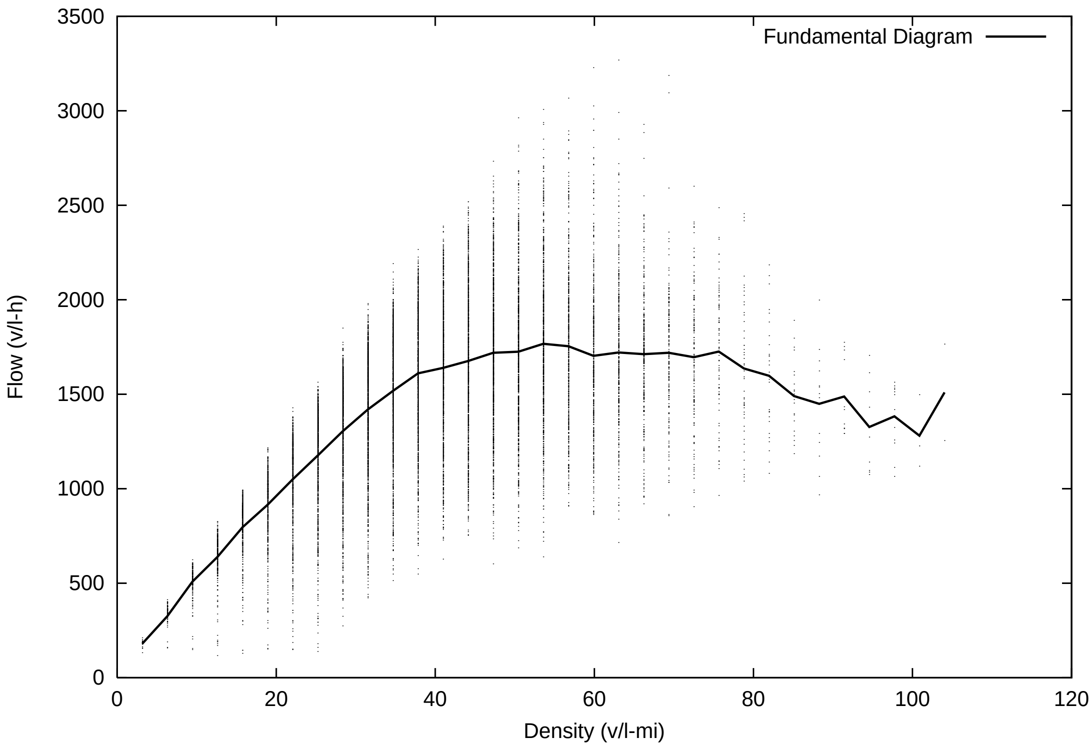

Figure 4: Fundamental Diagram, plotted with data from the Mulholland dataset. The line through the data was drawn by directly connecting the mean values of the points at each density level.

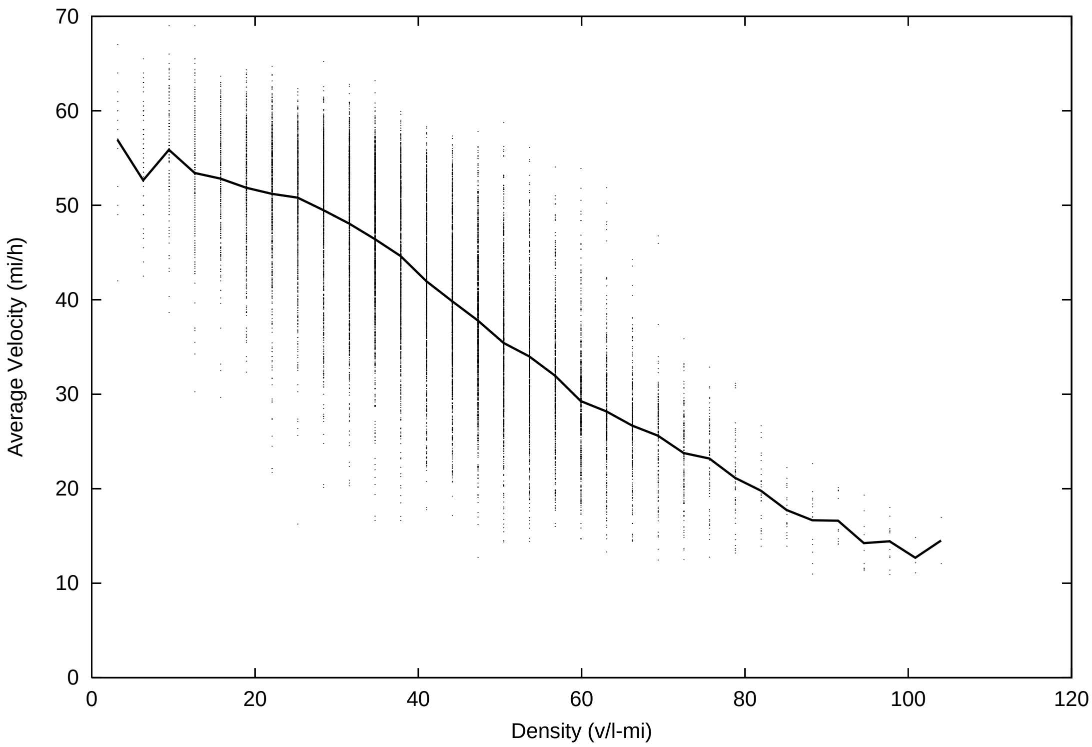

Figure 5: Velocity as a function of density. The line through the data was drawn by directly connecting the mean velocities of the points at each density level.

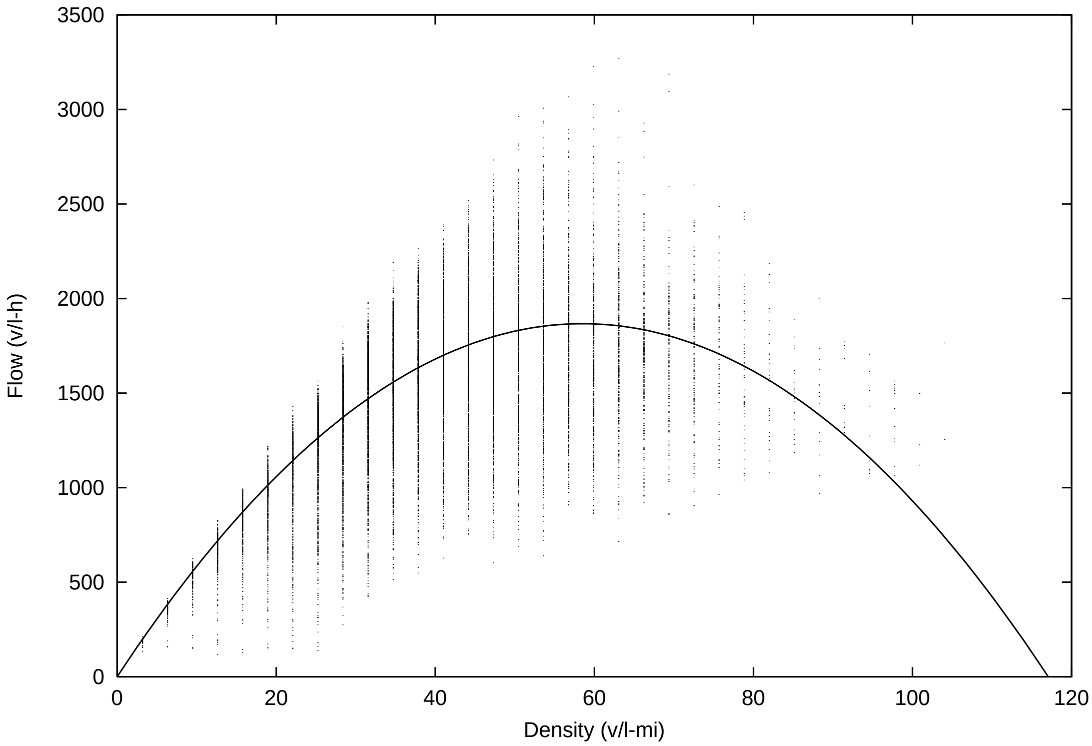

Figure 6: Modeling flow as a quadratic function of density.

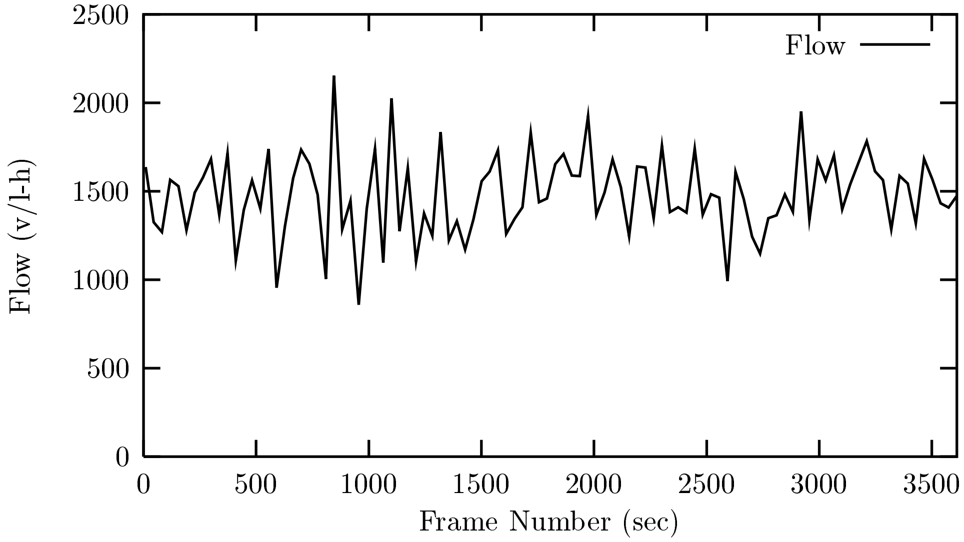

Figure 7: Flow is relatively constant, despite fluctuations in density and velocity.

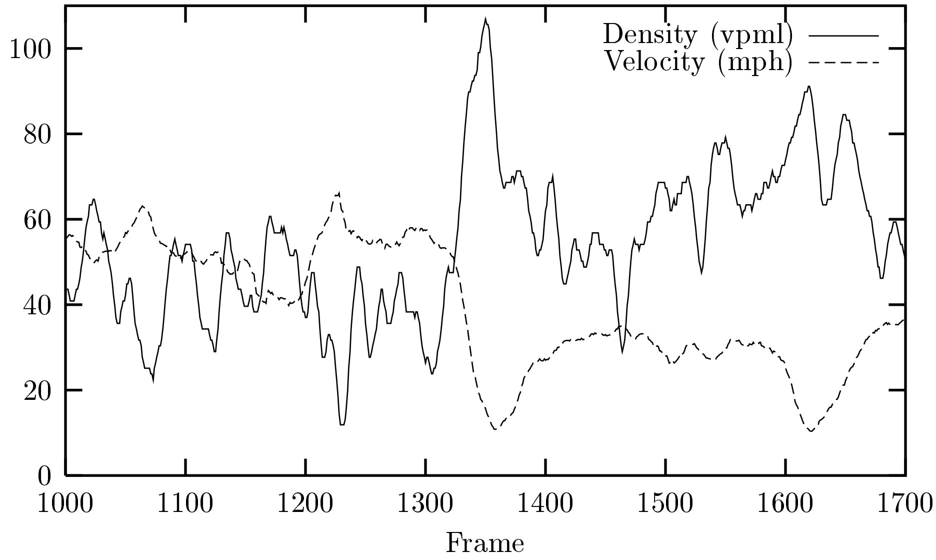

Figure 8: Local estimates of the density and velocity at each time point in lane 5, averaged over the 10 prior frames.

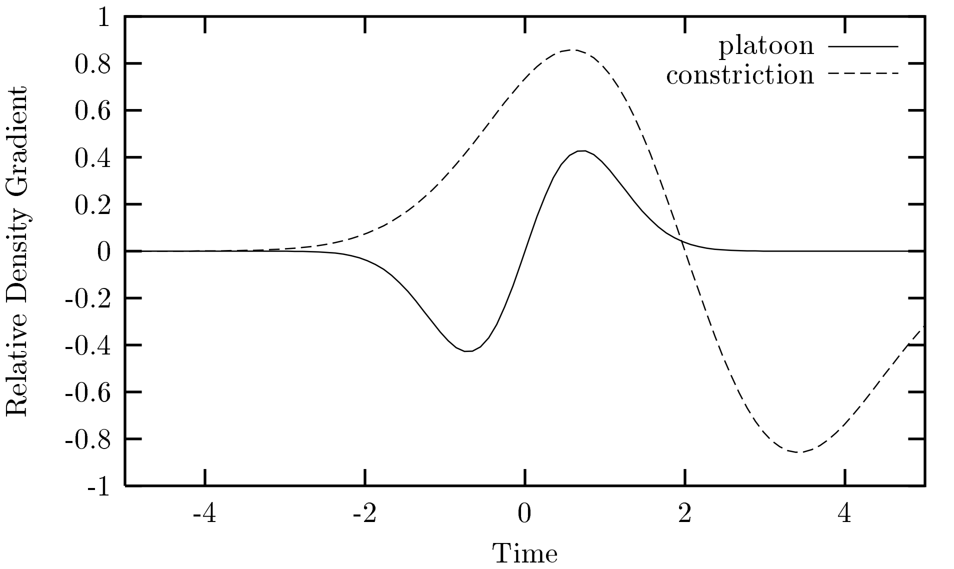

Figure 9: Expected form of gradient curve (horizontal axis is time), representing local difference in number of vehicles downstream from the midpoint of the section versus upstream, for: a) forward-moving platoons, and b) backward-moving constrictions.

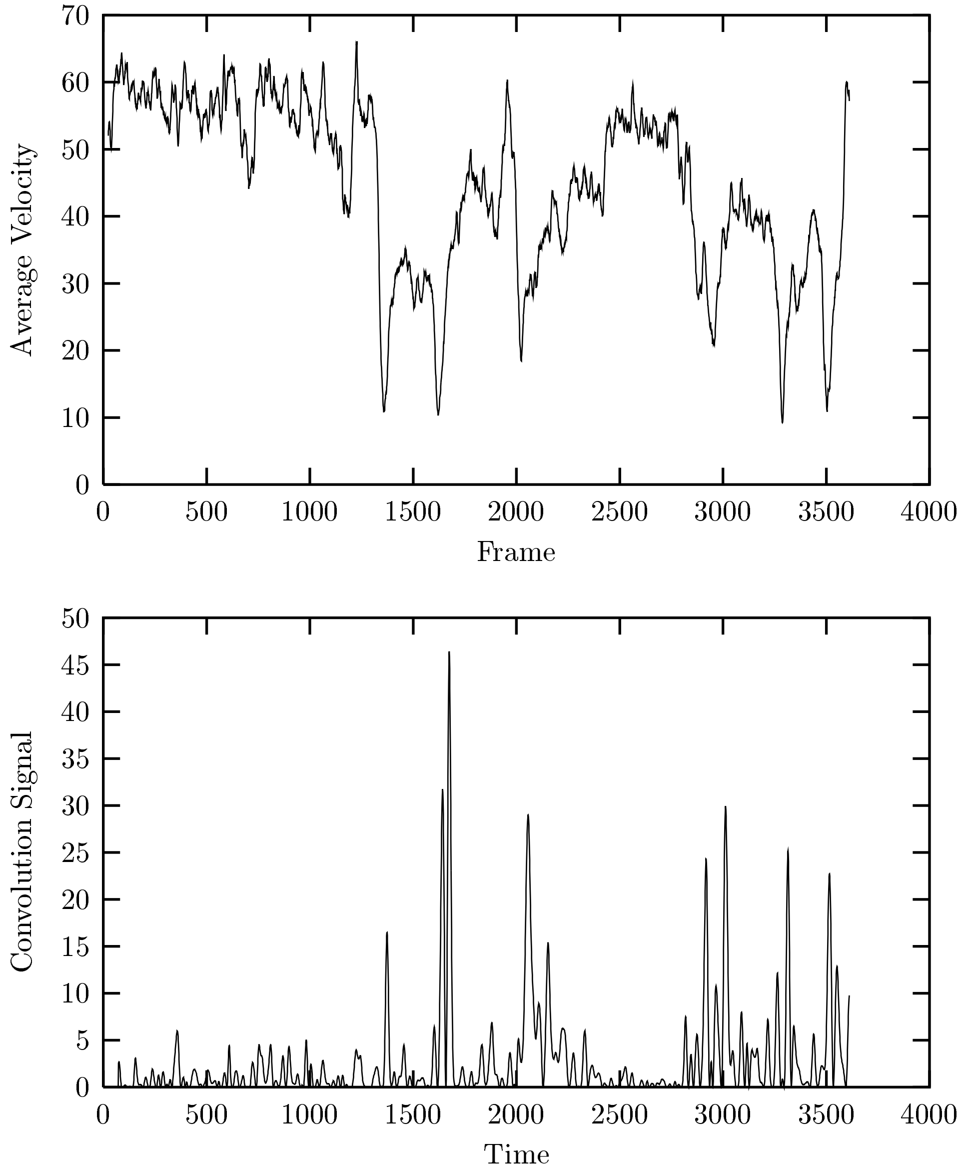

Figure 10: Local velocities at the midpoint of the section for lane 5 over all 3600 frames, and the signal detected by the convolution method.