Problem Set 9  
CPSC 411 Analysis of Algorithms  
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The assignment is due Thursday, Dec 03, 2009, before class.

**Exercise 1.** Let $G = (V, E)$ be an undirected graph, $V'$ a subset of $V$. Denote by $G_c$ the complementary graph of $G$, i.e., $G_c$ has vertex set $V$ and contains precisely the edges that are missing in $G$. Show that $V'$ is a clique in $G$ if and only if $V - V'$ is a vertex cover of $G_c$. [Hint: We stated this theorem in class but did not fully prove it.]

**Exercise 2.** Exercise 34.5-5 on page 1017 of our textbook.

**Exercise 3.** The SET COVER problem is the following decision problem: Given a family of sets $S_1, S_2, \ldots, S_n$ and an integer $k$ in the range $1 \leq k \leq n$. Does there exist a subfamily of $k$ sets $S_{i_1}, S_{i_2}, \ldots, S_{i_k}$ such that

$$
\bigcup_{j=1}^{k} S_{i_j} = \bigcup_{j=1}^{n} S_j
$$

Show that SET COVER is NP-complete. Use a polynomial reduction from VERTEX COVER to SET COVER. [Hint: Reductions do not need to be complicated! Try to re-interpret the VERTEX COVER problem in terms of sets.]

**Exercise 4.** Let us construct an undirected graph $G = (V, E)$ as follows. The nodes $V$ are the courses offered by a university during the current semester. There is an edge $\{u, v\}$ in $E$ if and only if the courses $u$ and $v$ have at least one student in common. The decision problem FINALS is: Given the graph $G$ and an integer $k$, does there exist $k$ or more courses whose finals can be scheduled at the same time? Show that FINALS is NP-complete. [Hint: Clique is NP-complete]

**Exercise 5.** Prove that the set $\mathbb{Z}$ of integers is countable by explicitly giving a bijective function $f : \mathbb{Z} \rightarrow \mathbb{N}$.

**Exercise 6.** Let $S$ be the set of infinite binary sequences. Show that the set $S$ is uncountable.