The assignment is due next Monday, Feb 2, 2009, before class.

The limit and limit superior criteria give in Lecture 2 for determining whether \( f(n) = O(g(n)) \) can be convenient in some of the subsequent exercises.

**Exercise 1.** Suppose that 
\[
p(n) = a_0 + a_1 n + \cdots + a_m n^m
\]
is a polynomial of degree \( m \) with complex coefficients. Show that 
\[
p(n) = O(n^k)
\]
for all \( k \geq m \), and 
\[
p(n) \neq O(n^\ell)
\]
for \( 0 \leq \ell < m \).

**Exercise 2.** Show that for fixed \( k \), we have 
\[
\binom{n}{k} = \frac{n^k}{k!} + O(n^{k-1}) \quad \text{and} \quad \binom{n+k}{k} = \frac{n^k}{k!} + O(n^{k-1}),
\]
where \( \binom{n}{k} = \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!} \) is the binomial coefficient.

**Exercise 3.** Show that 
\[
n/(n+1) = 1 + O(1/n).
\]

**Exercise 4.** Prove or disprove: 
\[
e^n = O(2^n).
\]

**Remark.** Let \( n_0 \) and \( n_1 \) be positive integers with \( n_1 > n_0 \). If 
\[
L(n_0) \geq R(n_0)
\]
holds, and the numbers \( L(n), R(n), L(n+1), \) and \( R(n+1) \) are positive and 
\[
\frac{L(n+1)}{L(n)} \geq \frac{R(n+1)}{R(n)}
\]
holds for all \( n \) in the range \( n_0 \leq n < n_1 \), then it follows that 
\( L(n) \geq R(n) \) holds for all \( n \) in the range \( n_0 \leq n \leq n_1 \).

**Exercise 5.** (a) Prove by induction on \( k \) that 
\[
\left(1 + \frac{1}{n}\right)^k < 1 + \frac{k}{n} + \frac{k^2}{n^2}
\]
holds for all \( k \) in \( \{1, 2\ldots, n\} \).
(b) Prove by induction that \( n! \geq (n/3)^n \) holds for all \( n \geq 1 \).
[HINT: Use part (a) of this exercise and the above remark].
(c) Deduce that \( \log(n!) = \Omega(n \log n) \) holds.

**Exercise 6 (CLRS).** Exercise 2.3-3, page 36