Problem Set 3

CPSC 411 Analysis of Algorithms

Andreas Klappenecker

The assignment is due next Wednesday, Feb 18, 2009, before class.

Exercise 1. *Let S be a finite set, k a positive integer, and*

$$F = \{A \mid A \subseteq S, |A| \leq k\}.$$

Show that (S, F) is a matroid.

Exercise 2. *Let $S = \{a, b, c, d\}$. Find the smallest matroid (S, F) such that F contains $\{a, b\}$ and $\{b, c, d\}$.*

Exercise 3. *Let $S = \{a, b, c, d\}$ and*

$$F = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}.$$

Suppose that the element have the weights $w(a) = 1$, $w(b) = 2$, $w(c) = 3$, and $w(d) = 4$. Show each step of the maximizing greedy algorithm Greedy((S,F),w). Show the order after sorting and the state of A after each iteration. Is the result correct?

Exercise 4. *Find the smallest example of an accessible set system (S, F) such that F contains two nonextendible sets A and B of different cardinality such that $A \subset B$. Explain why a smaller example cannot be found.*

Exercise 5. *Read the paper by Helman, Moret, and Shapiro* `http://www.cs.unm.edu/~moret/hms_IPCO.ps` *(and study Chapter 16 in [CLRS]).*

Give an exposition of their Theorem 1 and the proof of $(2) \Longrightarrow (3)$, *so that it becomes accessible to readers without much background. You have to formulate it in your own words and fill in all gaps. Neatly type this part of your homework. The correctness and the readability/quality of your exposition will be judged.*