$$\frac{1}{k+1}\left((n+1)^{k+1}-n^{k+1}\right) = \frac{1}{k+1}\left(\sum_{i=0}^{k+1}\binom{k+1}{i}n^i - n^{k+1}\right)$$

$$\leq \sum_{i=0}^{k}\binom{k+1}{i}n^i$$

$$\leq (\text{some constant})\, n^k$$