By Pascal's rule for binomial coefficients, we have

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}.$$

Therefore,

$$\Delta\binom{n}{k+1} = \binom{n}{k}.$$

In other words,

$$\sum \binom{n}{k}\,\delta n = \binom{n}{k+1}.$$

For example, this shows that

$$\sum_{n=0}^{m} \binom{n}{k} = \binom{m+1}{k+1} - \binom{0}{k+1} = \binom{m+1}{k+1}.$$