To solve our motivating example, we need to find a closed form for the sum

$$\sum_{k=1}^{n} k^2.$$

Since $k^2 = k^{\underline{2}} + k^{\underline{1}}$, an antiderivative of k^2 is given by

$$\sum k^2 \, \delta k = \sum (k^{\underline{2}} + k^{\underline{1}}) \delta k = \frac{1}{3} k^{\underline{3}} + \frac{1}{2} k^{\underline{2}}.$$

Thus, the sum

$$\sum_{k=1}^{n} k^2 = \frac{1}{3} k^{\underline{3}} \Big|_{1}^{n+1} + \frac{1}{2} k^{\underline{2}} \Big|_{1}^{n+1} = \ldots = \frac{n(2n+1)(n+1)}{6}.$$