

Theorem. Let $f(n)$ be an antiderivative of $g(n)$.
Then

$$\sum_{n=a}^b g(n) = f(b+1) - f(a).$$

Proof. We have

$$\begin{aligned} \sum_{n=a}^b g(n) &= \sum_{n=a}^b \Delta f(n) \\ &= \sum_{n=a}^b (f(n+1) - f(n)) \\ &= \sum_{n=a+1}^{b+1} f(n) - \sum_{n=a}^b f(n) = f(b+1) - f(a). \end{aligned}$$