Problem Set 1

Due dates: Electronic submission of .tex and .pdf files of this homework is due on **9/4/2012 before 10:00am** on csnet.cs.tamu.edu, a signed paper copy of the pdf file is due on **9/4/2012** at the beginning of class.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____________________________

Problem 1. (10 points) Get familiar with LaTeX. Nicely typeset the definition of e.

[Incidentally, we recommend that you keep a LaTeX file with all definitions and important theorems that we learn in this class. This will help you to memorize the definitions, and will allow you to quickly access this information when solving homework problems.]

Solution.

Problem 2. (15 points) Typeset the definitions of Big Oh, Big Omega, and Big Theta. Comment on the difference in definitions given in the lecture and in the textbook. Does it make a difference for the analysis of the running time of an algorithm which definition you use?

Solution.

Problem 3. (15 points) In each of the following situations, decide whether $f = O(g)$, $f = \Omega(g)$, or both (in which case $f = \Theta(g)$).
(a) $f(n) = n^2$ and $g(n) = n \log n$.
(b) $f(n) = n!$ and $g(n) = 2^n$.
(c) $f(n) = n^{100}$ and $g(n) = 2^n$.
(d) $f(n) = \sqrt{n^2 + 1}$ and $g(n) = n/2$.
(e) $f(n) = \log_{10}(n)$ and $g(n) = \ln(n)$.
No proofs need to be given.

Solution.

Problem 4. (15 points) Use the lim, lim sup, or lim inf criteria to prove the following facts:
(a) $3n^2 + 5n + \log n = O(n^2)$.
(b) $n^2 + (1 + (-1)^n)n = O(n^2)$.
(c) $H_n = \Omega(\log n)$, where $H_n = 1 + 1/2 + 1/3 + \cdots 1/n$.

Problem 5. (15 points) Consider the task of searching a sorted array `a[1..n]` for a given element w. Show that any algorithm that accesses the array only via comparisons (that is, by asking questions of the form "is `a[i]` $\leq z$?"), must take $\Omega(\log n)$ steps.

Solution.

Problem 6. (15 points) Show that any array `a[1..n]` of integers can be sorted in $O(n + M)$ time, where

$$M = \max_i a[i] - \min_i a[i].$$

When M is small, this is linear time. Explain why the $\Omega(n \log n)$ lower bound does not apply in this case.

Solution.

Problem 7. (15 points) Give a $(2n-1)$ lower bound on the number of comparisons needed to merge two sorted lists $(a_1, a_2, \ldots, a_n)$ and $(b_1, b_2, \ldots, b_n)$ with $a_1 < a_2 < \cdots < a_n$ and $b_1 < b_2 < \cdots < b_n$. [Hint: Use a proof by contradiction, assuming that there exists an algorithm to solve the problem correctly using $2n-2$ or fewer comparisons. Use as an input $(1, 3, \ldots, 2n-1)$ and $(2, 4, \ldots, 2n)$, and argue that the algorithm cannot correctly perform the merging of these lists.]

I will allow that you explore the last problem in class together with your team, **but** the homework solution must be formulated by yourself.

Solution.

Discussions on piazza are always encouraged, especially to clarify concepts that were introduced in the lecture. However, discussions of homework problems on piazza should not contain spoilers. It is okay to ask for clarifications concerning homework questions if needed.

Checklist:
- ☐ Did you add your name?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted)
- ☐ Did you sign that you followed the Aggie honor code?
- ☐ Did you solve all problems?
- ☐ Did you submit (a) your latex source file and (b) the resulting pdf file of your homework?
- ☐ Did you submit (c) a hardcopy of the pdf file in class?