Problem Set 3

> **Due dates:** Electronic submission of .tex and .pdf files of this homework is due on **9/18/2013 before 10:00am** on csnet.cs.tamu.edu, a signed paper copy of the pdf file is due on **9/18/2013** at the beginning of class.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____________________________

Make sure that you describe all solutions in your own words, even though these exercises were part of team explorations!

Problem 1. (a) (10 points) Discuss Strassen's matrix multiplication algorithm in your **own** words What is the idea behind the algorithm?

(b) (20 points) Illustrate the first step in Strassen's recursive algorithm to form the matrix product using 4×4 matrices. [You do not need to recurse.] Use the algorithm given on the slides, not the one given in our textbook.

(c) (20 points) Prove that Strassen's algorithm is correct. Use induction. Key points: Make sure the correct result is established for each of the four quadrants. Use the algorithm given on the slides, not the one given in our textbook.

Solution.

Read Chapter 30.

Problem 2. (a) (10 points) Suppose that you are given a polynomial

$$A(x) = \sum_{k=0}^{n-1} a_k x^k.$$

The input to the FFT of length n is given by an array containing the coefficients $(a_0, \ldots, a_{n-1})$. Describe the output of the FFT in terms of the polynomial $A(x)$.

(b) (10 points) Let ω be a primitive nth root of unity. The fast Fourier transform implements the multiplication with the matrix

$$F = (\omega^{ij})_{i,j\ in[0..n-1]}.$$

Show that the inverse of the F is given by

$$F^{-1} = \frac{1}{n}(\omega^{-jk})_{j,k\ in[0..n-1]}$$

[Hint: $x^n - 1 = (x-1)(x^{n-1} + \cdots + x + 1)$, so any power $\omega^\ell \neq 1$ must be a root of $x^{n-1} + \cdots + x + 1$.] Thus, the inverse FFT, called IFFT, is nothing but the FFT using ω^{-1} instead of ω, and multiplying the result with $1/n$.

(c) (10 points) Describe how to do a polynomial multiplication using the FFT and IFFT for polynomials $A(x)$ and $B(x)$ of degree $\leq n-1$. Make sure that you describe the length of the FFT and IFFT needed for this task.

(d) (15 points) How can you modify the polynomial multiplication algorithm based on FFT and IFFT to do multiplication of long integers in base 10? Make sure that you take care of carries in a proper way.

(e) (5 points) Illustrate your method from the previous part by multiplying the numbers 6789 and 4567. [You can directly multiply the polynomials and skip the FFT and IFFT steps. The processing of the resulting polynomial is of interest here.]

Solution.

Discussions on piazza are always encouraged, especially to clarify concepts that were introduced in the lecture. However, discussions of homework problems on piazza should not contain spoilers. It is okay to ask for clarifications concerning homework questions if needed.

Checklist:

- ☐ Did you add your name?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted)
- ☐ Did you sign that you followed the Aggie honor code?
- ☐ Did you solve all problems?
- ☐ Did you submit (a) your latex source file and (b) the resulting pdf file of your homework?
- ☐ Did you submit (c) a hardcopy of the pdf file in class?