Problem Set 4

Due dates: Electronic submission of .tex and .pdf files of this homework is due on **9/25/2013 before 10:00am** on csnet.cs.tamu.edu, a signed paper copy of the pdf file is due on **9/25/2013** at the beginning of class.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____________________________

Make sure that you describe all solutions in your own words, even though most of these exercises were part of team explorations!

Read Chapter 16.4 and skim rest of the chapter.

Problem 1.

P 1. (15 points) Suppose that a country adopts coins of values 1, 7, and 10. Does the greedy algorithm to give change always give the fewest number of coins? Prove it or give the smallest counter example.

Solution.

P 2. (15 points) Suppose that a country adopts coins of values 1, 3, and 6. Does the greedy algorithm to give change always give the fewest number of coins? Prove it or give the smallest counter example.

Solution.

P 3. (10 points) Consider Kruskals algorithm for a graph $G = (V, E)$ with edges $\{1, 2\}$, $\{2, 3\}$, $\{3, 4\}$, $\{4, 1\}$, $\{2, 4\}$. If the weight of the edges are

$$w(\{1, 2\}) = 1, w(\{2, 3\}) = 5, w(\{3, 4\}) = 6, w(\{4, 1\}) = 2, w(\{2, 4\}) = 4.$$

In which order will Kruskal's algorithm pick the edges?

Solution.

P 4. (15 points) Let S be a finite set and k a positive integer. Prove that (S, U_k) is a matroid, where

$$U_k = \{A \mid A \subseteq S, |A| \leq k\}.$$

Solution.

P 5. (5 points) (P1 continued.) Given weight function $w \colon S \to \mathbf{R}_{\geq 0}$ from S to the nonnegative real numbers, what would the greedy algorithm return when used with the matroid (S, U_k)?

Solution.

P 6. (20 points) Exercise 16.4-4 on page 443 in our textbook.

Solution.

P 7. (20 points) Let S be a finite set, F a nonempty family of subsets of S that satisfies the hereditary axiom. Show that if (S, F) is **not** a matroid, that is, does not satisfy the exchange axiom, then there exists a weight function $w \colon S \to \mathbf{R}_{\geq 0}$ such that Greedy((S, F), s) does not return a maximum weight basis of F, (a basis is a set in F that is not contained in any larger set in F). [Hint: Consider two subsets A and B such that $|A| < |B|$ but such that there does not exist any $x \in B \setminus A$ satisfying $A \cup \{x\}$ in F. Assume that A has m elements and construct a weight w such that the algorithm will return a set that has weight $w(A)$ even though $w(A) < w(B)$.]

Solution.

Discussions on piazza are always encouraged, especially to clarify concepts that were introduced in the lecture. However, discussions of homework problems on piazza should not contain spoilers. It is okay to ask for clarifications concerning homework questions if needed.

Checklist:

☐ Did you add your name?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit (a) your latex source file and (b) the resulting pdf file of your homework?
☐ Did you submit (c) a hardcopy of the pdf file in class?