Problem Set 10

Due dates: Electronic submission of .tex and .pdf files of this homework is due on **11/23/2015 before 11:00am** on csnet.cs.tamu.edu, a signed paper copy of the pdf file is due on **11/23/2015** at the beginning of class.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: _____

Problem 1

P 1 (30 points)**.** Cantor's function $k\colon \mathbf{N}_0 \times \mathbf{N}_0 \to \mathbf{N}_0$ is defined as

$$k(x, y) = \frac{1}{2}(x + y + 1)(x + y) + y. \tag{1}$$

Show that

(a) the function $k(x, y) = T_n + y$, where $n = x + y$ denotes the diagonal and $T_n = 1 + 2 + \cdots + n$ is the triangular number,

(a) the function $k(x, y)$ is injective

(a) and surjective

[This provides the formal proof that $\mathbf{N}_0 \times \mathbf{N}_0$ is countable.]

Solution.

P 2 (10 points)**.** Show that the subset of a countable set is countable.

Solution.

P 3 (30 points)**.** Show that there cannot exist a program that will determine on input of a program A whether or not A will print the string "Computer science is cool!" after a finite number of steps. [Hint: Reduce the halting problem to this problem.]

Solution.

P 4 (30 points)**.** A company claims to have an implementation of a C++ compiler that will detect at compile time whether a variable is uninitialized before it is used. Is this claim credible?

Solution.

Discussions on piazza are always encouraged, especially to clarify concepts that were introduced in the lecture. However, discussions of homework problems on piazza should not contain spoilers. It is okay to ask for clarifications concerning homework questions if needed.

Checklist:
☐ Did you add your name?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit the pdf file resulting from your latex file of your homework?
☐ Did you submit (c) a hardcopy of the pdf file in class?