Problem Set 2

Due dates: Electronic submission of .pdf files of this homework is due on **9/16/2015 before 11:00am** on ecampus, a signed paper copy of the pdf file is due on **9/16/2015** at the beginning of class.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: ____________________________

As always: Watch posted videos before the next class.

Problem 1 (10 points)**.** Consider the following code to find the second largest element:

```
largest := numbers[0];
secondLargest := null
for i=1 to numbers.length-1 do
    number := numbers[i];
    if number > largest then
        secondLargest := largest;
        largest := number;
    else
        if number > secondLargest then
            secondLargest := number;
        end;
    end;
end;
```

This code was provided by someone in response to a question on stackoverflow. (a) How many comparisons does this code make, assuming that `numbers` contains n elements. (b) Give a small example which shows that this is not optimal.

Solution.

Problem 2 (10 points)**.** Describe an algorithm in pseudocode that finds the 2nd largest element in the least possible number of steps. Explain why your algorithm is correct and why it finds the 2nd largest element in the least possible number of steps.

Solution.

Problem 3. (15 points) Consider the task of searching a sorted array `a[1..n]` for a given element w. Show that any algorithm that accesses the array only via comparisons (that is, by asking questions of the form "is `a[i]` $\leq z$?"), must take $\Omega(\log n)$ steps.

Solution.

Problem 4. (15 points) Give a $(2n-1)$ lower bound on the number of comparisons needed to merge two sorted lists $(a_1, a_2, \ldots, a_n)$ and $(b_1, b_2, \ldots, b_n)$ with $a_1 < a_2 < \cdots < a_n$ and $b_1 < b_2 < \cdots < b_n$. [Hint: Use an adversarial method. Why can't you have in general $2n-2$ or fewer comparisons?]

Solution.

Problem 5. (10 points) Solve Exercise 8.1-4 on page 194 of our textbook.

Solution.

Problem 6 (20 points)**.** Exercise 4.5-1a, b, c, d on page 96

Solution.

Problem 7 (10 points)**.** Generalize Karatsubas integer multiplication algorithm to numbers with respect to a base $b \geq 2$.

Solution.

Problem 8 (10 points)**.** (a) Implement your generalized Karatsuba's integer multiplication algorithm for base $b = 10$ in C++ or Java and include the documented codes here. (b) Print out the intermediate results in human readable form for the multiplication of the inputs 1234 and 9867 using the generalized Karatsuba multiplication algorithm and include the trace here.

Solution.

Discussions on ecampus are always encouraged, especially to clarify concepts that were introduced in the lecture. However, discussions of homework problems on piazza should not contain spoilers. It is okay to ask for clarifications concerning homework questions if needed.

Checklist:
☐ Did you add your name?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit (a) the pdf file derived from your latex source file of your homework?
☐ Did you submit (b) a hardcopy of the pdf file in class?