Problem Set 2

Due dates: Electronic submission of .tex and .pdf files of this homework is due on **2/6/2015 before 11:00am** on e-campus (as a turnitin assignment), a signed paper copy of the pdf file is due on **2/6/2015** at the beginning of class.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: _____

Problem 1. (30 points) In each of the following situations, decide whether $f = O(g)$, $f = \Omega(g)$, or both (in which case $f = \Theta(g)$), and prove your results.
(a) $f(n) = n^3$ and $g(n) = n^2 \log n$.
(b) $f(n) = n!$ and $g(n) = 2^n$.
(c) $f(n) = n^{1000}$ and $g(n) = 2^n$.
(d) $f(n) = \sqrt{n^2 + 1}$ and $g(n) = n/2$.
(e) $f(n) = \log_{10}(n)$ and $g(n) = \ln(n)$.
(f) $f(n) = n \ln n$ and $g(n) = \ln n!$.

Solution.

Problem 2. (10 points) Use the definitions to prove the following facts.
(a) $H_n = O(\log n)$, where $H_n = 1 + 1/2 + 1/3 + \cdots 1/n$.
(b) $H_n = \Omega(\log n)$, where $H_n = 1 + 1/2 + 1/3 + \cdots 1/n$.
Hint: Compare with an integral!

Solution.

Problem 3. (15 points) Use the lim, lim sup, or lim inf criteria to prove the following facts:
(a) $3n^2 + (-1)^n n^2 + 5n + \log n = O(n^2)$.
(b) $n^2 + (1 + (-1)^n)n = n^2 + O(n^2)$.
(c) $(4 + \sin(n\pi/2))n^3 + \cos(n\pi/2)n^2 + \ln n = \Omega(n^3)$.

Solution.

Problem 4. (10 points) Let f_1, f_2, g_1, g_1 be functions from the natural numbers to the real numbers. Suppose that $f_1 = \Theta(g_1)$ and $f_2 = \Theta(g_2)$. Prove or disprove: $f_1 + f_2 = \Theta(g_1 + g_2)$.

Solution.

Problem 5. (15 points) (a) Let α be a real number in the range $\alpha < 1$. Show that

$$\exp(x^\alpha) \sim \exp((x+1)^\alpha)$$

holds. [Hint: The mean value theorem from calculus can be handy.]

(b) Does the same conclusion hold if $\alpha \geq 1$?

Solution.

Problem 6. (10 points) Show that $f(x) = g(x) + O(1)$ implies that $\exp(f(x)) \asymp \exp(g(x))$.

Solution.

Problem 7. (10 points) Show that $f \in O(g)$ and $f \notin o(g)$ does not imply that $f \in \Theta(g)$.

Solution.

I will allow that you explore some of the problems in class together with your team, **but** the homework solution must be formulated by yourself. Homeworks must be typeset in LaTeX.

Solution.

Checklist:

- ☐ Did you add your name?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted)
- ☐ Did you sign that you followed the Aggie honor code?
- ☐ Did you solve all problems?
- ☐ Did you submit (a) your latex source file and (b) the resulting pdf file of your homework?
- ☐ Did you submit (c) a hardcopy of the pdf file in class?