Problem Set 9

Due dates: Electronic submission of this homework is due on **Wednesday 4/13/2016 before 3:50pm** on ecampus, a signed paper copy of the pdf file is due on **4/13/2016** at the beginning of class.

Name: (put your name here)

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Signature: _____

Problem 1 (20 points)**.** Given a truth table of a boolean function f. Explain how to derive the disjunctive normal form (a disjunction of clauses consisting of conjunctions of literals) of f. (a) Illustrate using $f(x, y, z) = x \oplus y \oplus z$ as an example. Here $\oplus$ denotes exlusive or. (b) Prove your result.

Solution.

Problem 2 (20 points)**.** Given a truth table of a boolean function f. Explain how to derive the conjunctive normal form (which is a conjunction of clauses consisting of disjunctions of literals) of f. (a) Illustrate using $f(x, y, z) = x \oplus y \oplus z$ as an example. (b) Prove your result.

Solution.

Problem 3 (50 points)**.** Given the formula f in 2-conjunctive normal form

$$f(x, y, z) = (\neg x \vee y) \wedge (\neg y \vee z) \wedge (x \vee \neg z) \wedge (z \vee y)$$

(a) Determine the implication graph of f and visualize it (e.g. using tikz)
(b) Determine the strongly connected components of the implication graph
(c) Visualize the acylic strongly connected component graph of the implication graph
(d) Assign truth values according to the algorithm given in the lecture on 2SAT. Is f satisfiable or not?
(e) Give a truth table for f.

Solution.

Problem 4 (10 points)**.** In the randomized algorithm for 2SAT, we used Markov's inequality, which states that a nonnegative random variable satisfies for every real number $b \geq \mathrm{E}[X]$ the bound

$$\Pr[X \geq b] \leq \frac{\mathrm{E}[X]}{b}.$$

Prove this bound.

Solution.

Discussions on piazza are always encouraged, especially to clarify concepts that were introduced in the lecture. However, discussions of homework problems on piazza should not contain spoilers. It is okay to ask for clarifications concerning homework questions if needed.

Checklist:
☐ Did you add your name?
☐ Did you disclose all resources that you have used?
   (This includes all people, books, websites, etc. that you have consulted)
☐ Did you sign that you followed the Aggie honor code?
☐ Did you solve all problems?
☐ Did you submit (a) your latex source file and (b) the resulting pdf file of your homework?
☐ Did you submit (c) a hardcopy of the pdf file in class?