

Example 3. Suppose that we have a boolean function $f: \mathbf{F}_2^n \rightarrow \mathbf{F}_2$. A quantum circuit implementing f has to be realized by a unitary map. This can be accomplished, for instance, by implementing the map

$$|y\rangle \otimes |x\rangle \mapsto |y \oplus f(x)\rangle \otimes |x\rangle$$

on $n + 1$ qubits, where $x \in \mathbf{F}_2^n$, and $y \in \mathbf{F}_2$. The most significant bit is the output bit, and the n lowest significant bits are the input bits. The result of $f(x)$ is added modulo 2 to the output bit. The result is a quantum circuit of