

Let us denote by $S(b)$ and $R(c)$ the matrices

$$S(b) = \begin{pmatrix} e^{-ib} & 0 \\ 0 & e^{ib} \end{pmatrix} \quad \text{and} \quad R(c) = \begin{pmatrix} \cos c & -\sin c \\ \sin c & \cos c \end{pmatrix}.$$

The statement of the previous lemma is that a unitary matrix can be written in the form $U = e^{ia}S(b)R(c)S(d)$ for some $a, b, c, d \in \mathbf{R}$. Notice that

$$XR(c)X = R(-c) \quad \text{and} \quad XS(b)X = S(-b).$$