

Proof. We can write U in the form $U = e^{ia}V$, where V is some unitary matrix with determinant 1. The matrix V has to be of the form $V = \begin{pmatrix} \alpha & -\bar{\beta} \\ \beta & \bar{\alpha} \end{pmatrix}$. Indeed, the columns of a unitary matrix are orthogonal, hence the right column of V has to be a multiple of $(-\bar{\beta}, \bar{\alpha})^t$; and the determinant constraint forces V to be of the given form. We can write α and β in the form $\alpha = e^{ih} \cos c$ and $\beta = e^{-ik} \sin c$ for some real numbers h, k, c , because α and β satisfy $|\alpha|^2 + |\beta|^2 = 1$; it follows that

$$V = \begin{pmatrix} e^{ih} \cos c & -e^{ik} \sin c \\ e^{-ik} \sin c & e^{-ih} \cos c \end{pmatrix}.$$