Let B denote the unitary map on $\mathbf{C}^4$ determined by (3.4). We will derive the solution in some small steps. It is clear that we have to take advantage of the superposition principle to evaluate the boolean function simultaneously for both possible input arguments. The solution to Deutsch's problem uses an additional trick, which allows us to encode the value of $f(x)$ into a phase factor. Suppose that the least significant bit is in the state $1/\sqrt{2}(|0\rangle - |1\rangle)$, then

$$B\left(|x_1\rangle \otimes \big(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\big)\right) = |x_1\rangle \otimes \big(\frac{1}{\sqrt{2}}|f(x_1)\rangle - \frac{1}{\sqrt{2}}|1 \oplus f(x_1)\rangle\big) =: v_{x_1}$$

for all $x_1 \in \{0, 1\}$. If the value of $f(x_1)$ is zero, then the input state remains invariant; otherwise, B affects a change of sign. Explicitly,

$$v_{x_1} = (-1)^{f(x_1)}|x_1\rangle \otimes \big(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\big).$$