Proof. The gate on the left hand side acts on basis states in the following way:

$$\begin{aligned}
|00\rangle \otimes |x\rangle &\mapsto |00\rangle \otimes |x\rangle \\
|01\rangle \otimes |x\rangle &\mapsto |01\rangle \otimes |x\rangle \\
|10\rangle \otimes |x\rangle &\mapsto |10\rangle \otimes |x\rangle \\
|11\rangle \otimes |x\rangle &\mapsto |11\rangle \otimes U|x\rangle
\end{aligned}$$