By the Cayley-Hamilton theorem, the unitary 2×2 matrix U satisfies its characteristic equation $U^2 + (\operatorname{tr} U)U + (\det U)I = 0$; thus,

$$(\operatorname{tr} U)U = U^2 + (\det U)I.$$

Using this relation, we obtain

$$\begin{aligned} V^2 &= \frac{1}{\operatorname{tr} U \pm 2\sqrt{\det U}}(U \pm \sqrt{\det U}\, I)^2 \\ &= \frac{1}{\operatorname{tr} U \pm \sqrt{\det U}}(U^2 + (\det U)I \pm 2\sqrt{\det U}\, U) \\ &= \frac{1}{\operatorname{tr} U \pm 2\sqrt{\det U}}(\operatorname{tr} U \pm 2\sqrt{\det U})U = U \end{aligned}$$