Problem Set 3
CPSC 440/640 Quantum Algorithms
Andreas Klappenecker

The assignment is due on Wednesday, Monday, October 2.
Demonstrate your program during my office hours.

The goals of this assignment are (a) to make you familiar with the simulation of quantum circuits on a classical computer, and (b) to give you the opportunity to get familiar with lex and yacc (use flex and bison if possible).

You are given a considerable amount of time for this assignment so that this exercise does not provide a conflict with your research. I recommend that you start early. You can earn 200 points instead of the usual 100 points. Some source code is provided in the tar-ball `alfred.tgz`.

1) The core simulator is contained in the file `sim.c`. Supplement the missing code in the procedures `measure_state` and `applygate`.

2) Write a small test file that uses the procedures given in `sim.c` to create the state $0.707|00) + 0.707|11)$ from input state $|00)$. Do this by simulating the action of one Hadamard gate and one controlled-not gate with `applygate`. This should be followed by measuring the state with `measure_state`. Monitor the evolution of the state after each step by `print_state`.

3) Get familiar with lex and yacc, or, rather, with flex and bison. Read the manuals and implement some small example that allows you to grasp the main concept of the interaction between lex and yacc.

4) Supplement the missing code in `alfred.y` and correct all errors so that you get a fully functional simulator for the language Alfred.

You will receive a little Alfred program and you need to demonstrate that your simulator works. The details will be discussed in class.

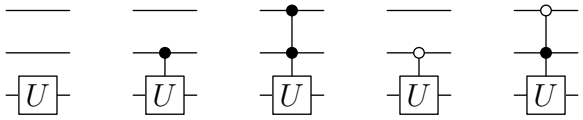
Remarks. i) The simulator assumes that you will not simulate more than 20-30 quantum bits. For efficiency reasons, information about the position of control and target qubits are encoded by setting the corresponding bits in an integer. For example, if the target qubit position `pos` is the least significant bit, then this is represented by `pos = 1<<0`, if the target qubit is the most significant qubit in a system of 3 qubits, then this is encoded by `pos = 1<<2`. Review the C bit operations to see the benefit of this convention.

ii) Write the code for the procedure `measure_state`. The input for this procedure is an integer `pos`, which has a bit set at the position of the quantum bit, which will be observed. The measurement is done with respect to the computational basis. Your procedure should directly modify the input state

vector `state`. You can address the content of this state vector by `state[`i`]`, where `0` $\leq i <$ `1<<Nbits`. Use the random number generator `rand` in your implementation. Make sure that your implementation will reflect the behaviour of quantum mechanics. You might need to change the initialization of the random number generator on you system to obtain the desired results.

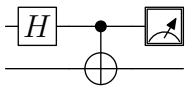
iii) Write the code for the procedure `applygate`. Your code should realize an implementation of a multiply conditioned gate, as explained in the lecture. The conditions are provided in terms of integers. A bit set in `ocnd` means that this bit must be 0, a bit set in `icnd` means that this bit must be set to 1. The integer `gpos` has a single bit set at the target bit position of the gate.

iv) Before writing the code for `applygate`, I suggest that you write down the 8×8 matrices realizing the gates:



Note that these matrices are sparse, so you do not want to implement a full matrix multiplication to realize these gates. Two basis states are affected by a small 2×2 matrix. Try to understand the action of the gates on the basis states $|000), ..., |111)$. Study the corresponding bit patterns for the various versions of the above gates. What does change if the target bit is chosen differently?

v) The circuit that you should implement in question 2) is given by



vi) Use the GNU gcc compiler. The source code uses some language extensions provided by gcc. If you use some Windows operating system, then I suggest that you have a look at cygwin, which provides you with all the Unix utilities such as lex and yacc,...