Intersecting Simple Surfaces

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Types of Surfaces

- Infinite Planes
- Polygons
  - Convex
  - Ray Shooting
  - Winding Number
- Spheres
- Cylinders
Infinite Planes

- Defined by a **unit** normal $n$ and a point $o$

  $$n \cdot (x - o) = 0$$
Infinite Planes

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  $L(t) = p + v t$
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$$n \cdot (x - o) = 0$$

$$L(t) = p + vt$$

$$n \cdot (p + vt - o) = 0$$
**Infinite Planes**

- Defined by a **unit** normal $n$ and a point $o$

\[ n \cdot (x - o) = 0 \]

\[ L(t) = p + vt \]

\[ n \cdot vt = n \cdot (o - p) \]
Infinite Planes

- Defined by a unit normal \( n \) and a point \( o \)

\[
n \cdot (x - o) = 0
\]

\[
L(t) = p + vt
\]

\[
t = \frac{n \cdot (o - p)}{n \cdot v}
\]
Infinite Planes

- Defined by a **unit** normal $n$ and a point $o$

\[ n \cdot (x - o) = 0 \]

\[ L(t) = p + vt \]

\[ p + v \frac{n \cdot (o - p)}{n \cdot v} \]
Polygons

- Intersect infinite plane containing polygon
- Determine if point is inside polygon
Polygons

- Intersect infinite plane containing polygon
- Determine if point is inside polygon

- How do we know if a point is inside a polygon?
Point Inside Convex Polygon
Point Inside Convex Polygon

- Check if point on same side of all edges
Point Inside Convex Polygon

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\[
\begin{vmatrix}
N^T \\
(P_i - X)^T \\
(P_{i+1} - X)^T
\end{vmatrix}
\text{ must be same sign}
\]
Point Inside Polygon Test

- Given a point, determine if it lies inside a polygon or not
Ray Test

- Fire ray from point
- Count intersections
  - Odd = inside polygon
  - Even = outside polygon
Problems With Rays

- Fire ray from point
- Count intersections
  - Odd = inside polygon
  - Even = outside polygon
- Problems
  - Ray through vertex
Problems With Rays

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Problems With Rays

- Fire ray from point
- Count intersections
  - Odd = inside polygon
  - Even = outside polygon
- Problems
  - Ray through vertex
  - Ray parallel to edge
A Better Way
A Better Way
A Better Way
A Better Way
A Better Way
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A Better Way
A Better Way
A Better Way
A Better Way

- One winding = inside
A Better Way
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- zero winding = outside
Requirements

- Oriented edges
- Edges can be processed in any order
Computing Winding Number

- Given unit normal $n$
- $\theta = 0$
- For each edge $(p_1, p_2)$

$$\theta^+ = \frac{n \cdot ((p_1 - x) \times (p_2 - x))}{|(p_1 - x) \times (p_2 - x)|} \cos^{-1}\left(\frac{(p_1 - x) \cdot (p_2 - x)}{|p_1 - x||p_2 - x|}\right)$$

- If $|\theta| > \pi$, then inside
Advantages

- Extends to 3D!
- Numerically stable
- Even works on models with holes (sort of)
- No ray casting
Intersecting Spheres

- Three possible cases
  - Zero intersections: miss the sphere
  - One intersection: hit tangent to sphere
  - Two intersections: hit sphere on front and back side

- How do we distinguish these cases?
Intersecting Spheres

\[ F(x) = (x - c) \cdot (x - c) - r^2 = 0 \]
Intersecting Spheres

\[ F(x) = (x - c) \cdot (x - c) - r^2 = 0 \]

\[ F(L(t)) = (p + vt - c) \cdot (p + vt - c) - r^2 = 0 \]
Intersecting Spheres

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\[ F(L(t)) = (p + vt - c) \cdot (p + vt - c) - r^2 = 0 \]

\[ F(L(t)) = (v \cdot v)t^2 + 2v \cdot (p - c)t + (p - c) \cdot (p - c) - r^2 = 0 \]
Intersecting Spheres

- $F(L(t)) = 0$ is quadratic in $t$

$$F(L(t)) = (v \cdot v)t^2 + 2v \cdot (p - c)t + (p - c) \cdot (p - c) - r^2 = 0$$
Intersecting Spheres

- $F(L(t)) = 0$ is quadratic in $t$

\[ F(L(t)) = (v \cdot v)t^2 + 2v \cdot (p - c)t + (p - c) \cdot (p - c) - r^2 = 0 \]

- Solve for $t$ using quadratic equation

\[ t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

- If $b^2 - 4ac < 0$, no intersection

- If $b^2 - 4ac = 0$, one intersection

- Otherwise, two intersections
Normals of Spheres

\[ F(x) = (x - c) \cdot (x - c) - r^2 = 0 \]

\[ \nabla F(x) = x - c \]
Infinite Cylinders

- Defined by a center point $C$, a unit axis direction $A$ and a radius $r$
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Infinite Cylinders

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1. Perform an orthogonal projection to the plane defined by $C$, $A$ on the line $L(t)$ and intersect with circle in 2D
Infinite Cylinders

- Defined by a center point $C$, a unit axis direction $A$ and a radius $r$

2. Substitute $t$ parameters from 2D intersection to 3D line equation
Infinite Cylinders

- Defined by a center point $C$, a unit axis direction $A$ and a radius $r$

3. Normal of 2D circle is the same normal of cylinder at point of intersection
Infinite Cylinders

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Infinite Cylinders

- Defined by a center point $C$, a unit axis direction $A$ and a radius $r$

\[
N = \frac{P - C - ((P - C) \cdot A)A}{r}
\]