Intersecting Simple Surfaces

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TAMU Computer Science

Types of Surfaces

- Infinite Planes
- Polygons
  - Convex
  - Ray Shooting
  - Winding Number
- Spheres
- Cylinders

Infinite Planes

- Defined by a **unit** normal n and a point o

$$n \cdot (x - o) = 0$$

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$$n \cdot (p + vt - o) = 0$$

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$$n \cdot vt = n \cdot (o - p)$$

Infinite Planes

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$$t = \frac{n \cdot (o - p)}{n \cdot v}$$

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$$p + v \frac{n \cdot (o - p)}{n \cdot v}$$

Polygons

- Intersect infinite plane containing polygon
- Determine if point is inside polygon

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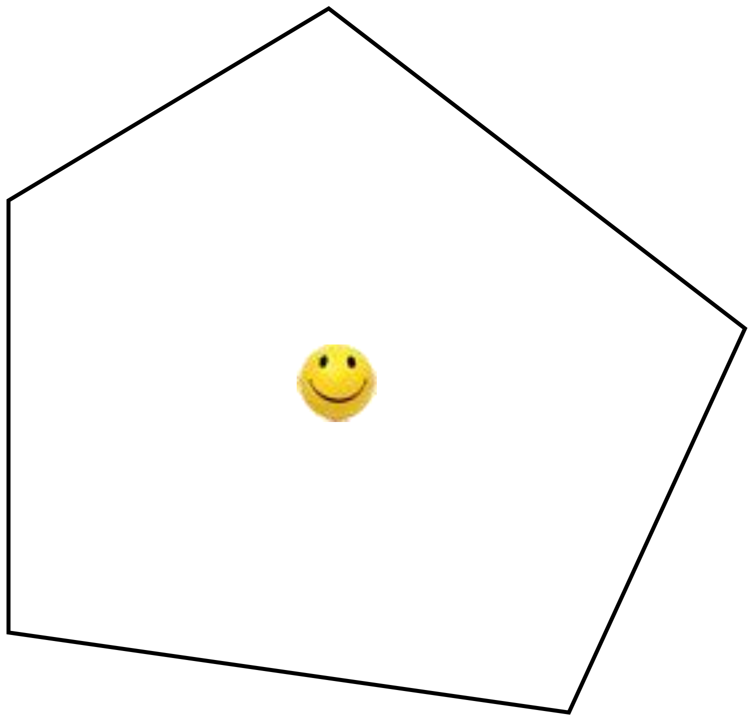
- Determine if point is inside polygon

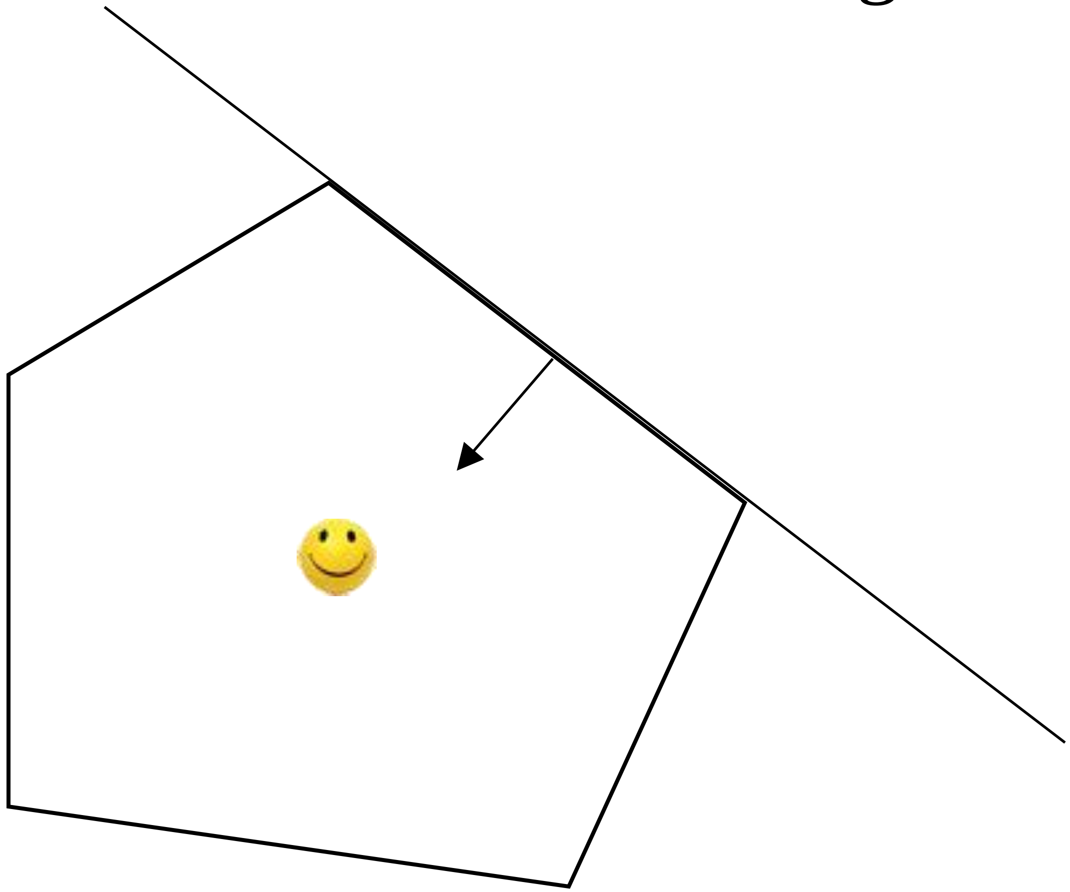
- How do we know if a point is inside a polygon?

Point Inside Convex Polygon



Point Inside Convex Polygon

- Check if point on same side of all edges



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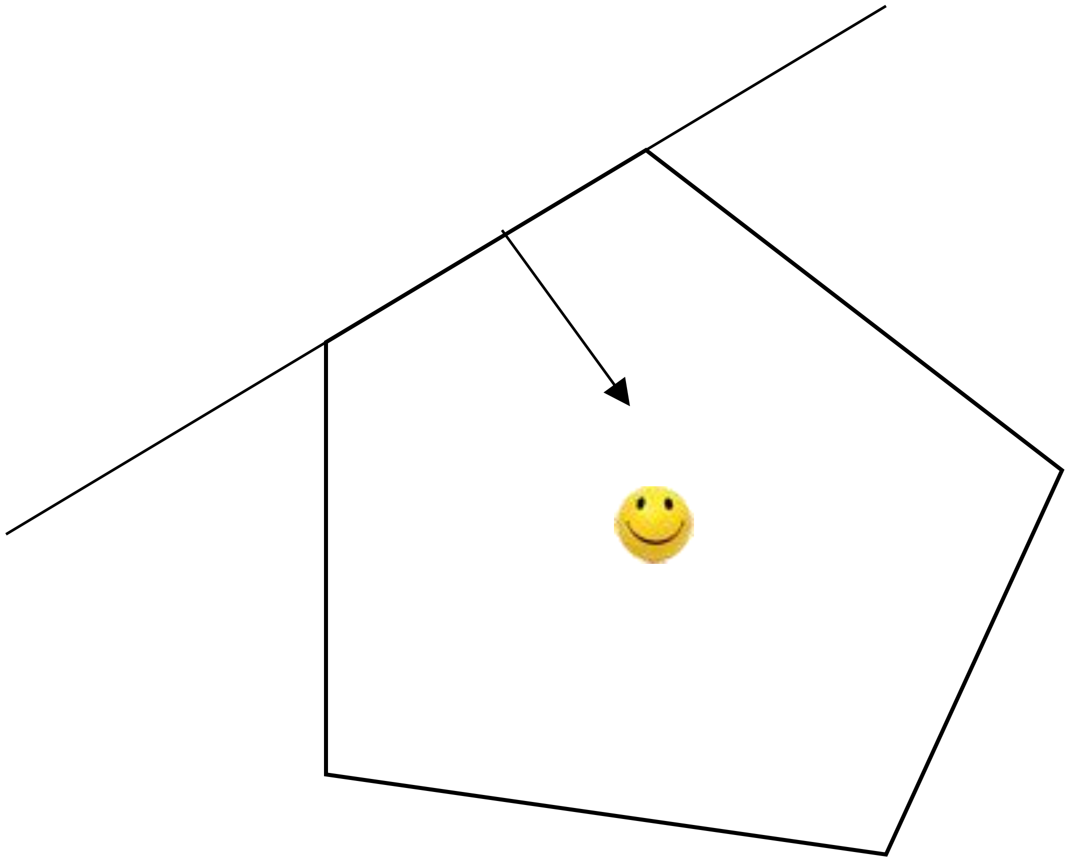

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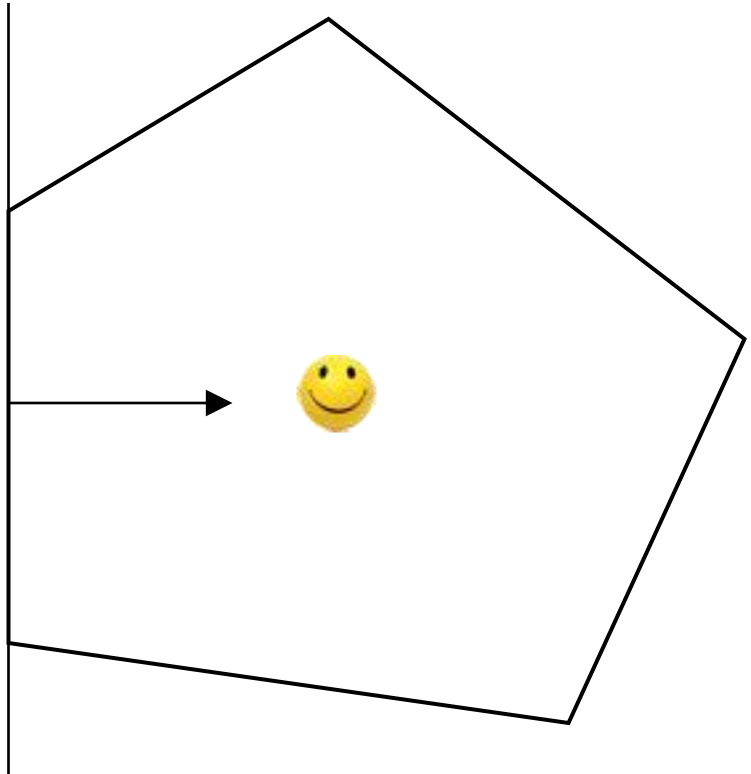

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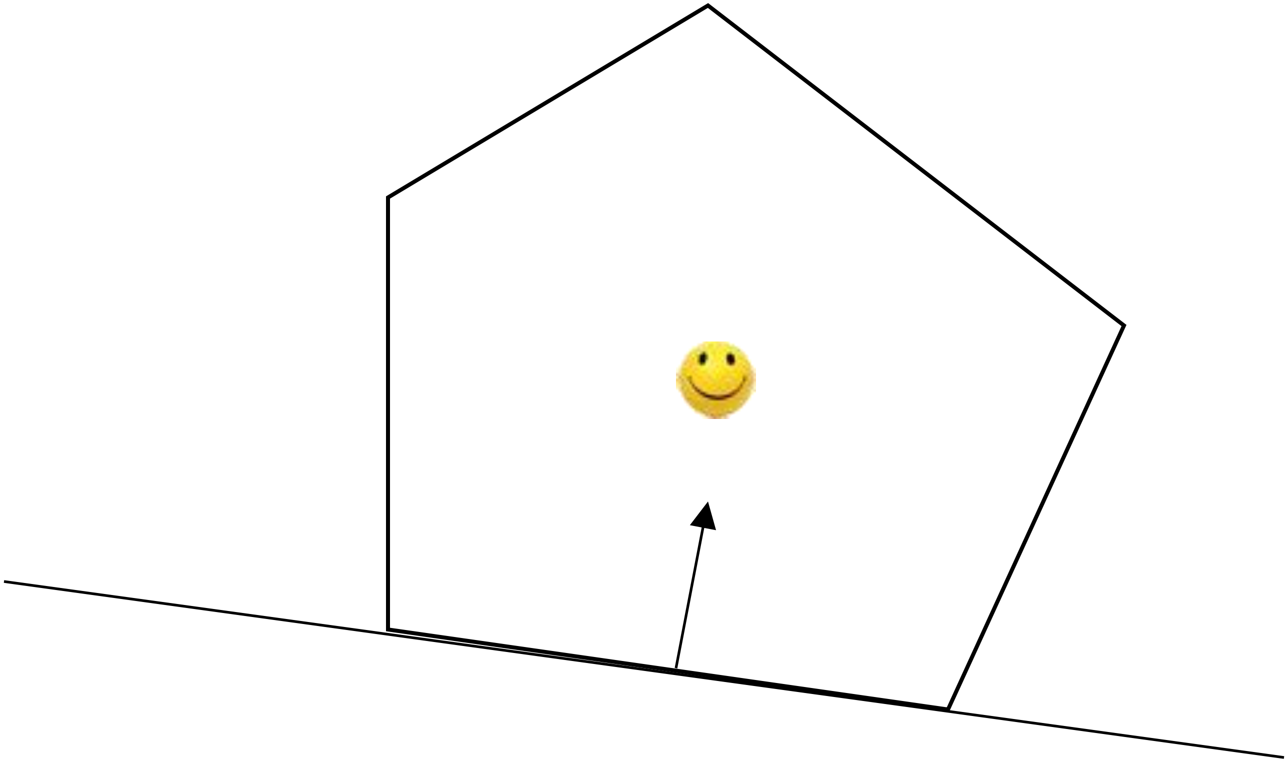

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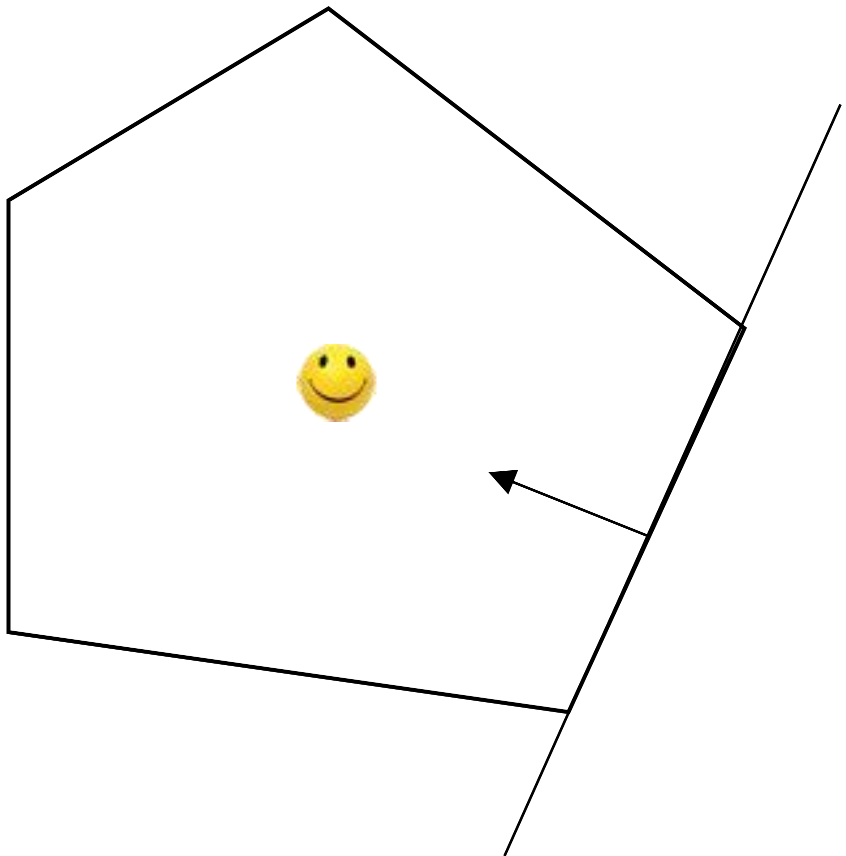

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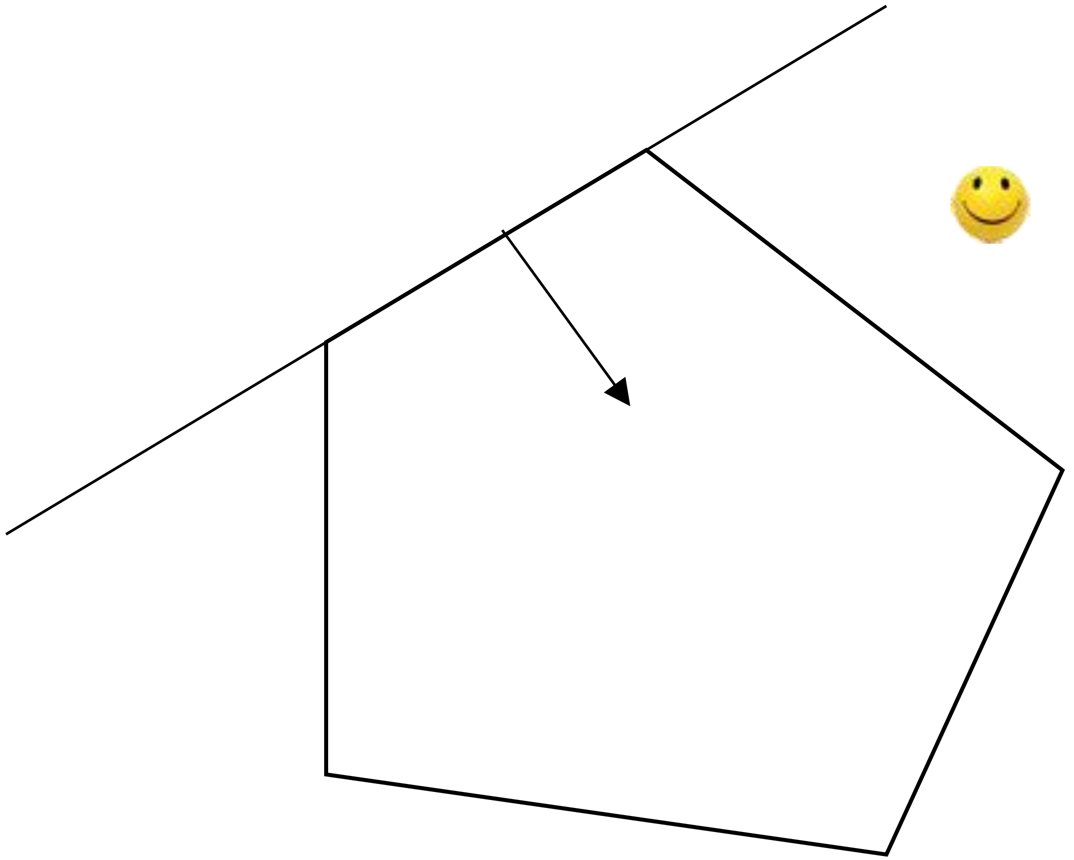

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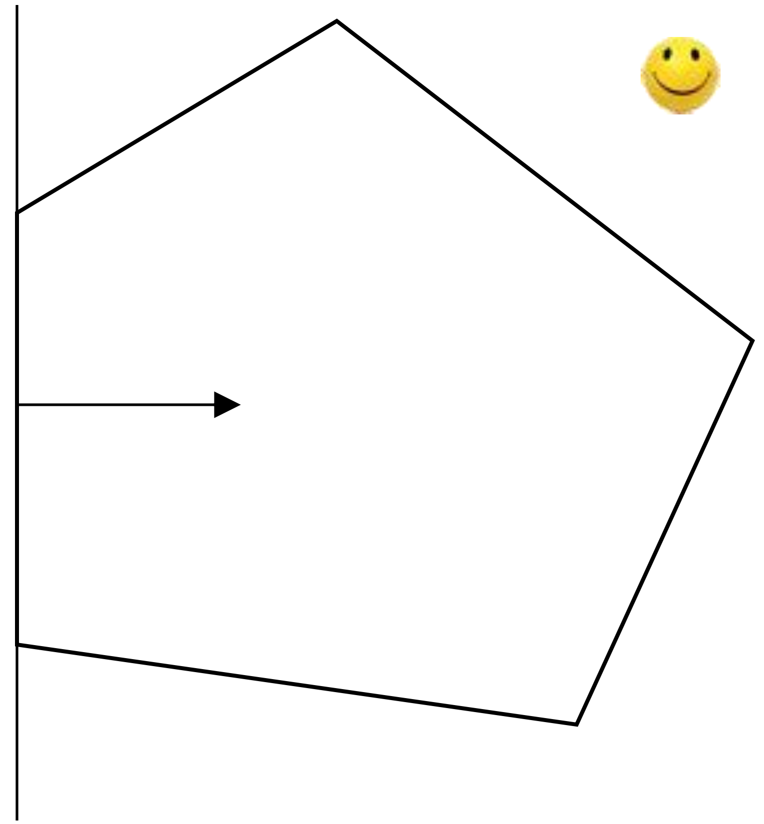

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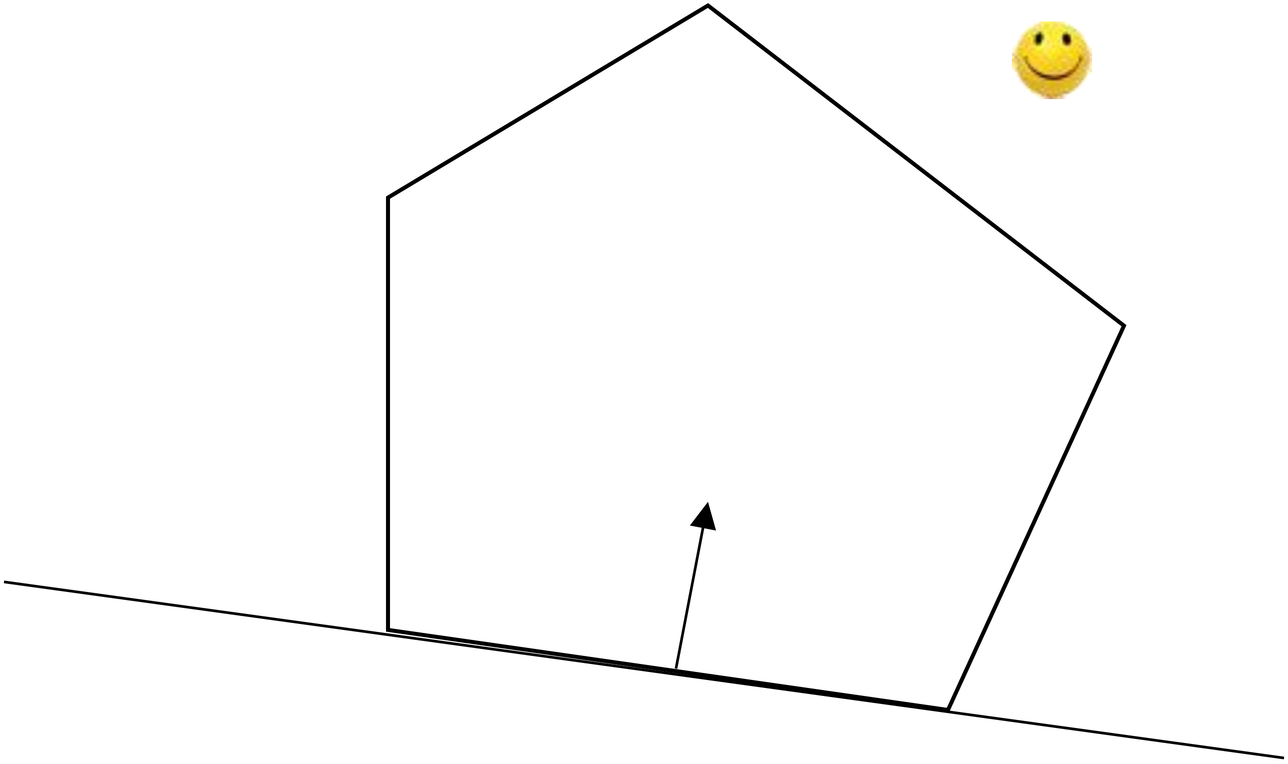

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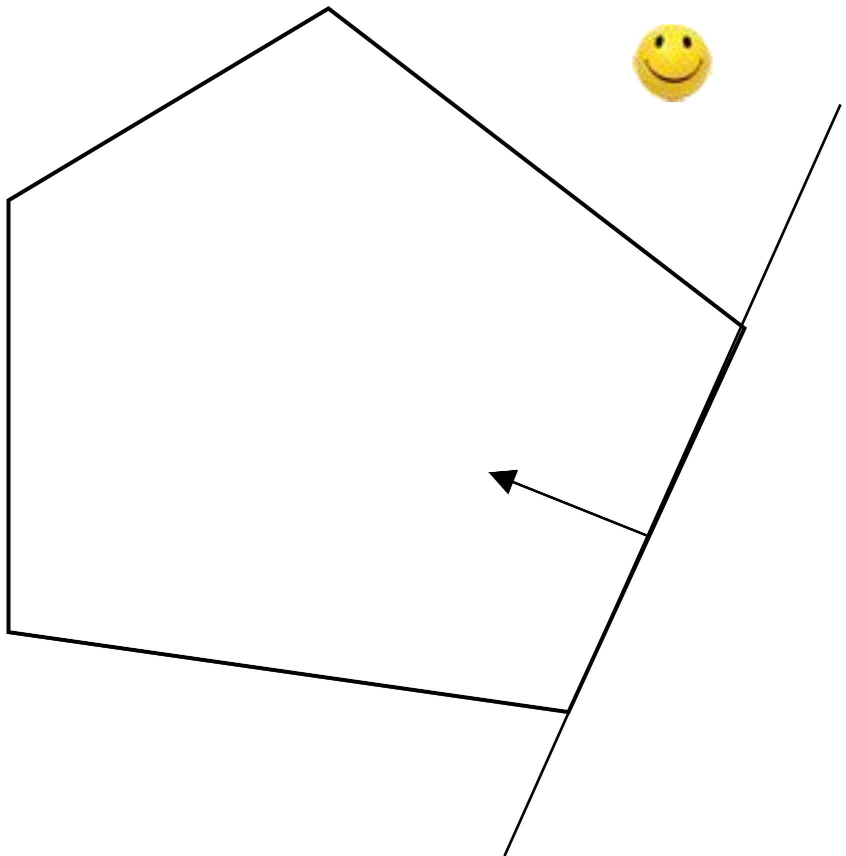

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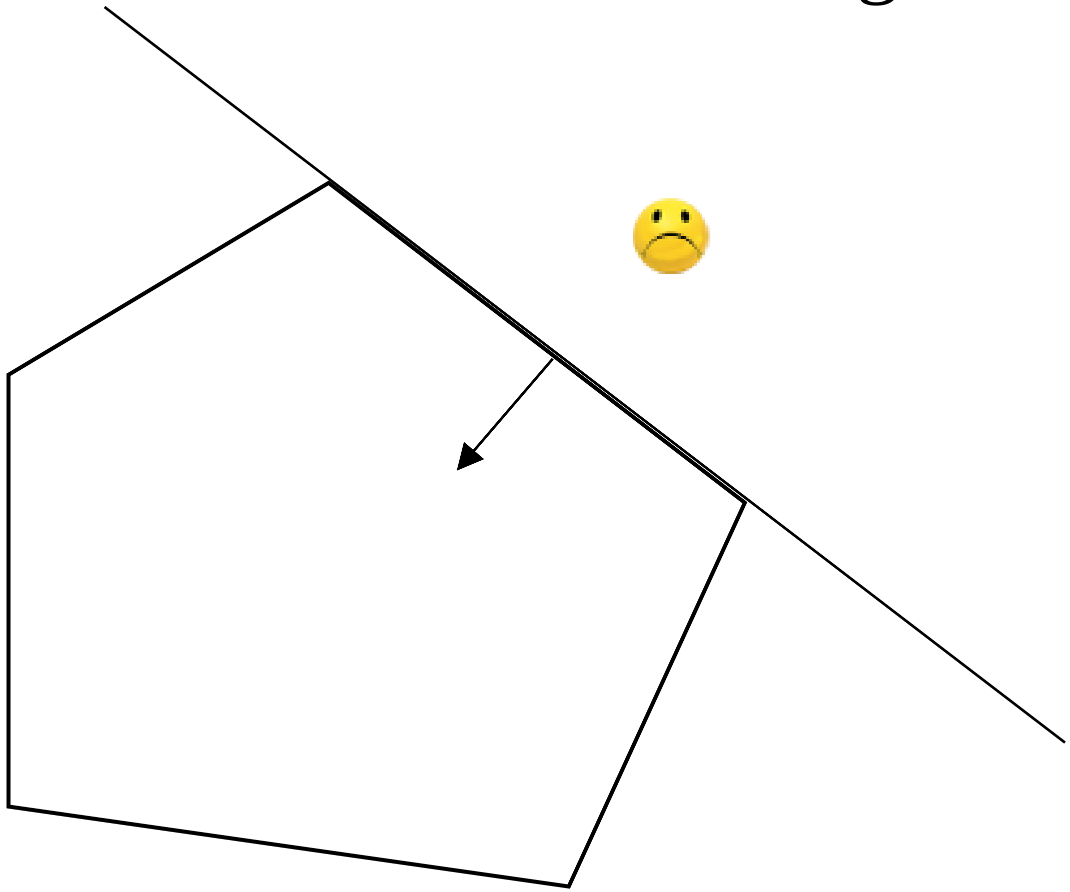

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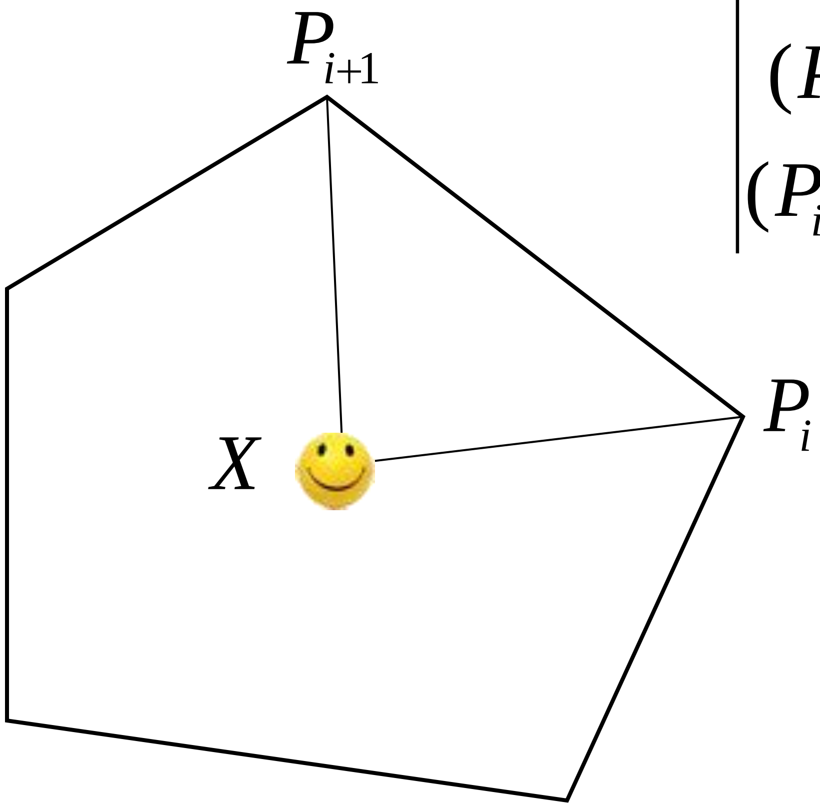
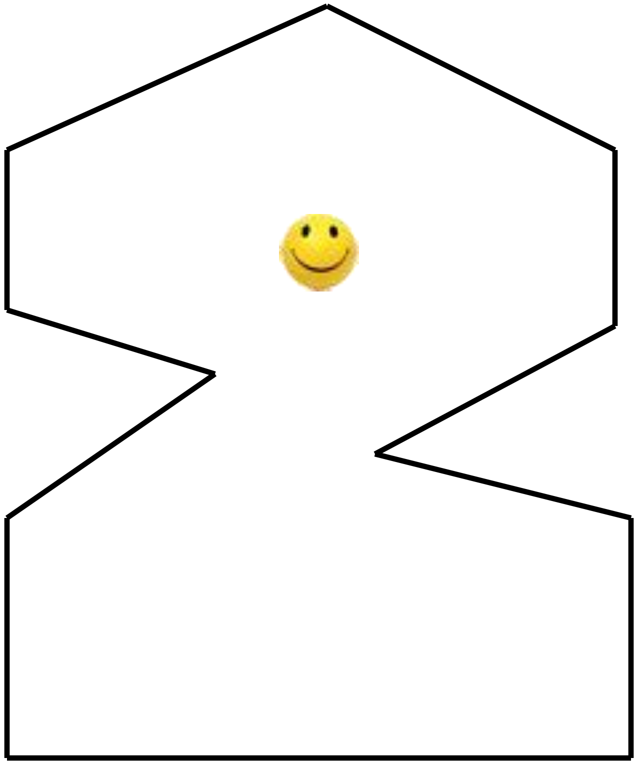


$$\begin{vmatrix} N^T \\ (P_i - X)^T \\ (P_{i+1} - X)^T \end{vmatrix} \text{ must be same sign}$$

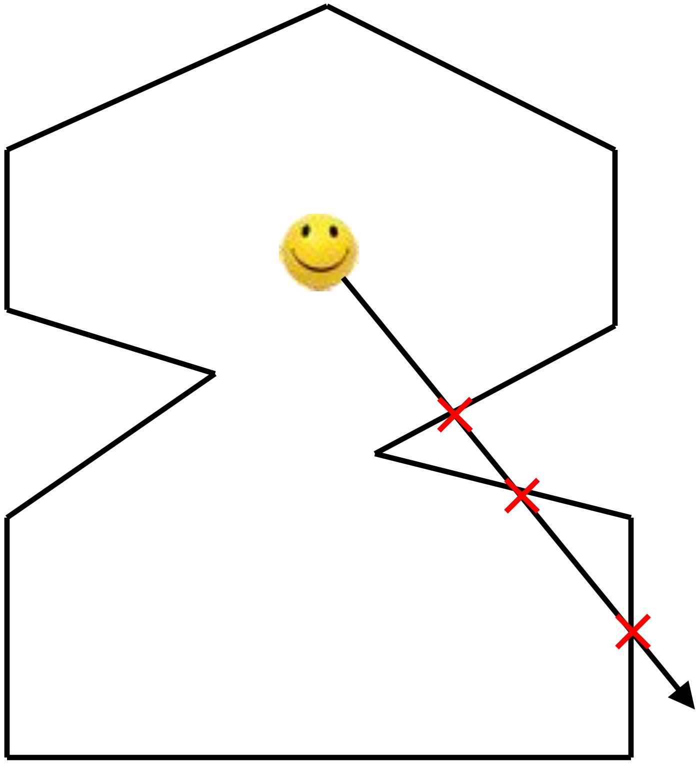
Point Inside Polygon Test

- Given a point, determine if it lies inside a polygon or not



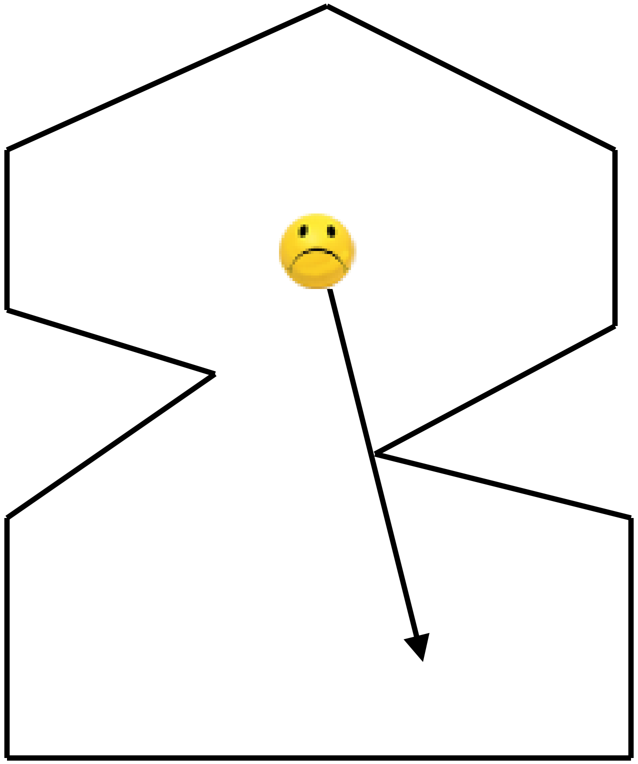
Ray Test

- Fire ray from point
- Count intersections
  - Odd = inside polygon
  - Even = outside polygon



Problems With Rays

- Fire ray from point
- Count intersections
  - Odd = inside polygon
  - Even = outside polygon
- Problems
  - Ray through vertex



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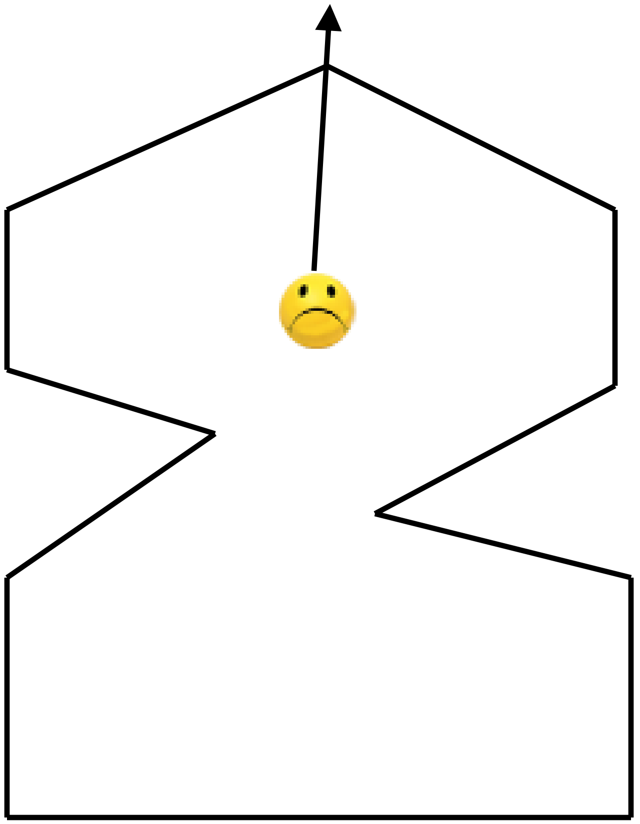
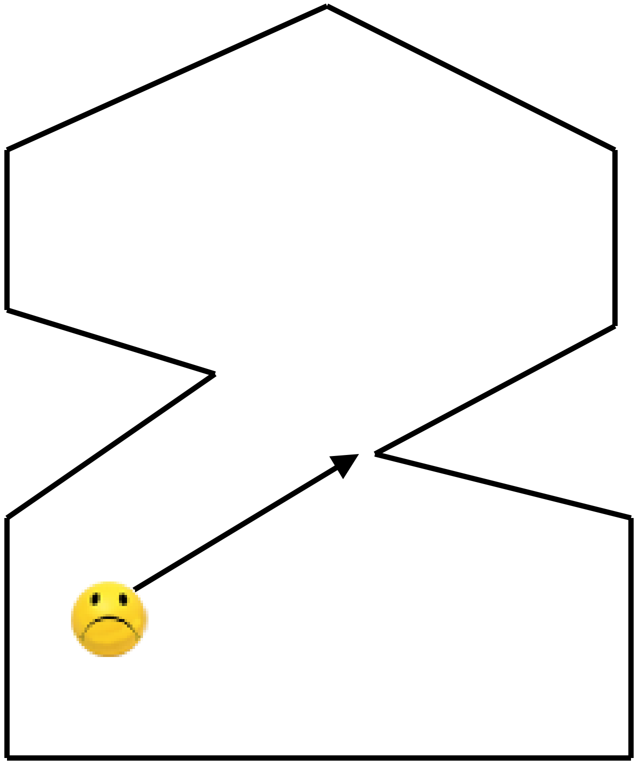
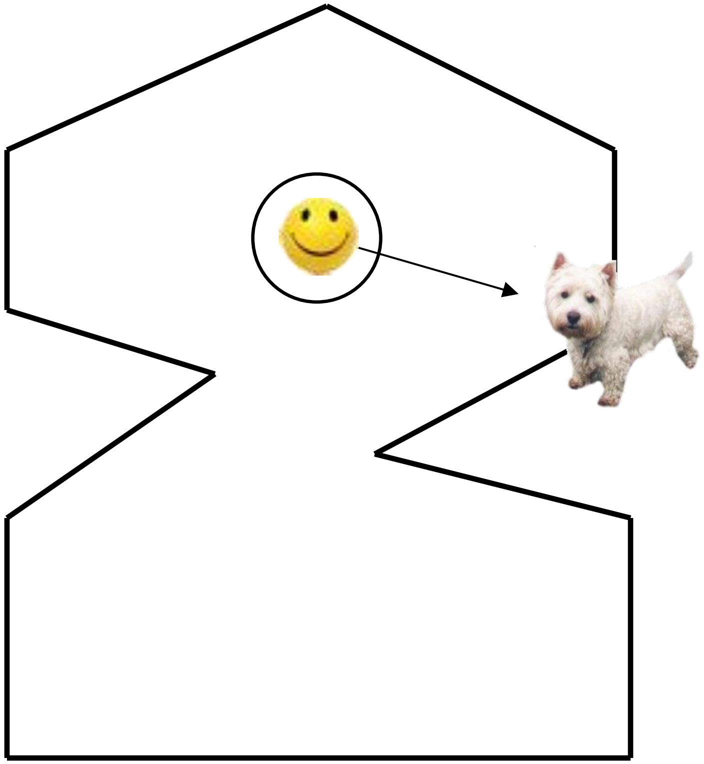


Problems With Rays

- Fire ray from point
- Count intersections
  - Odd = inside polygon
  - Even = outside polygon
- Problems
  - Ray through vertex
  - Ray parallel to edge



A Better Way



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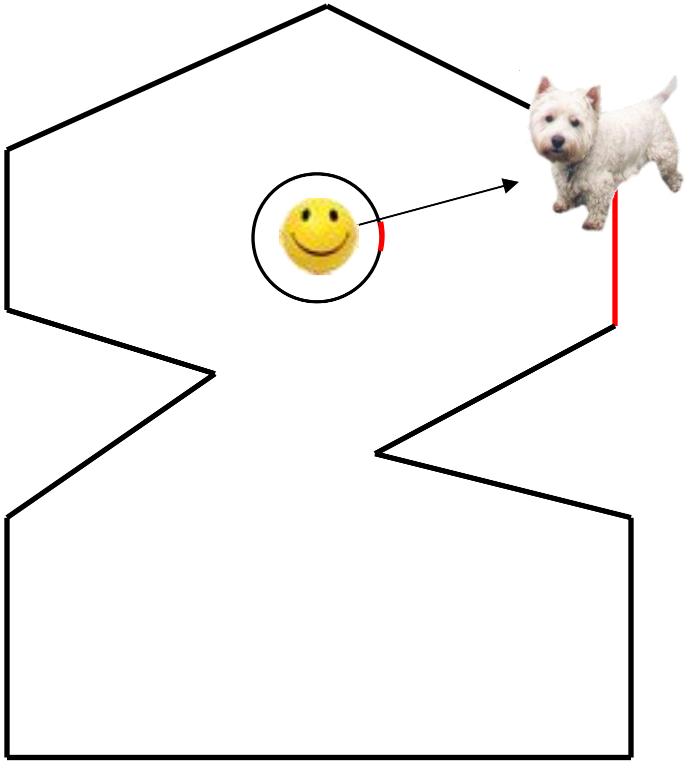

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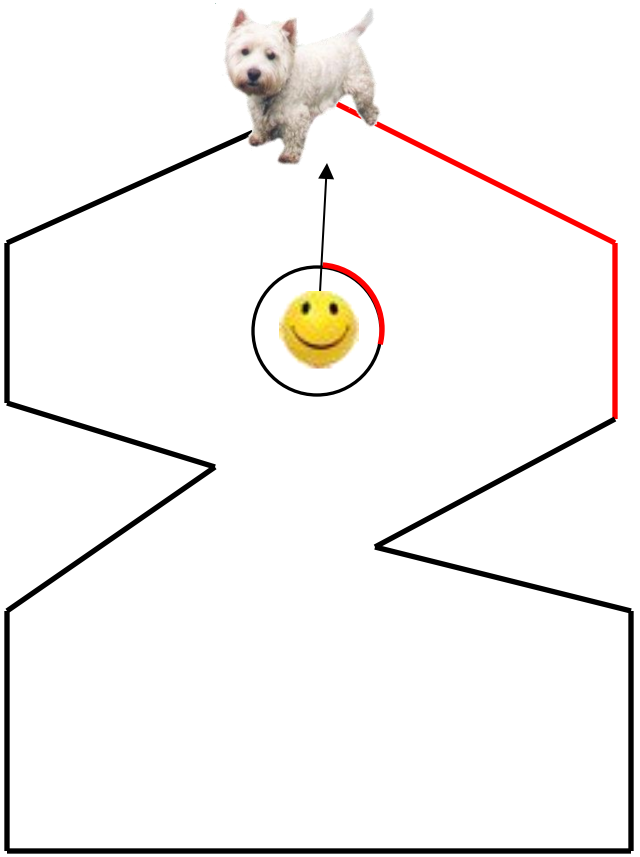

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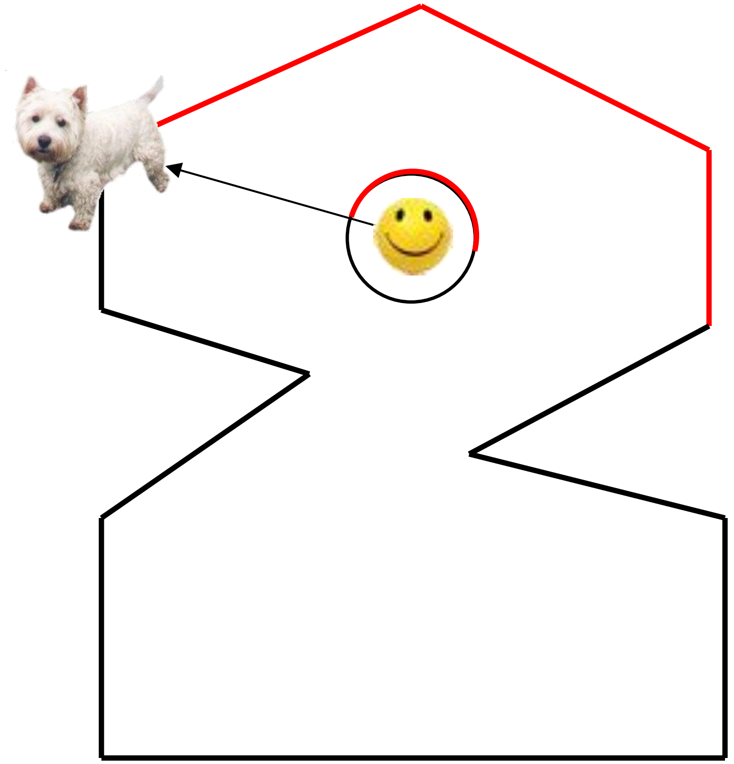

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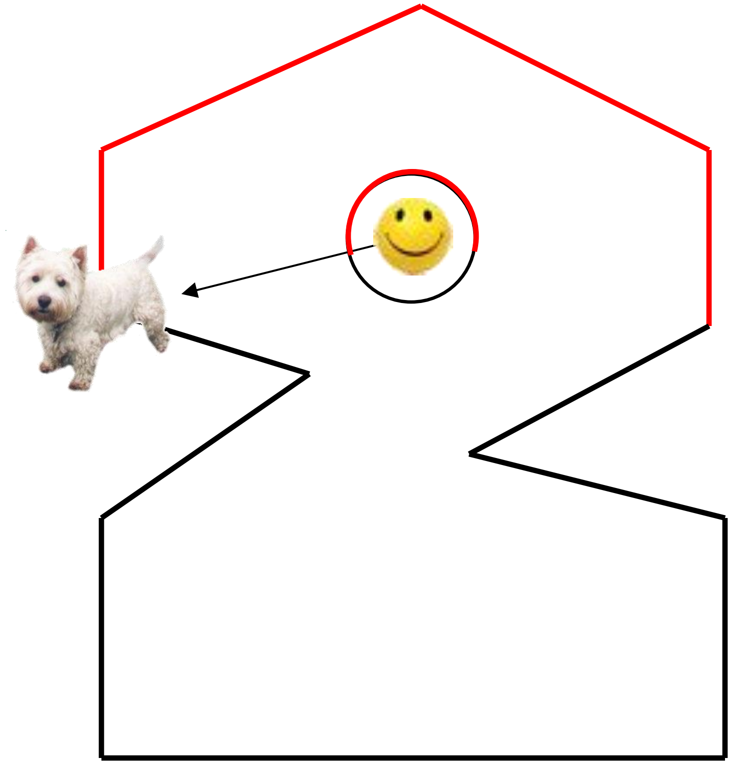

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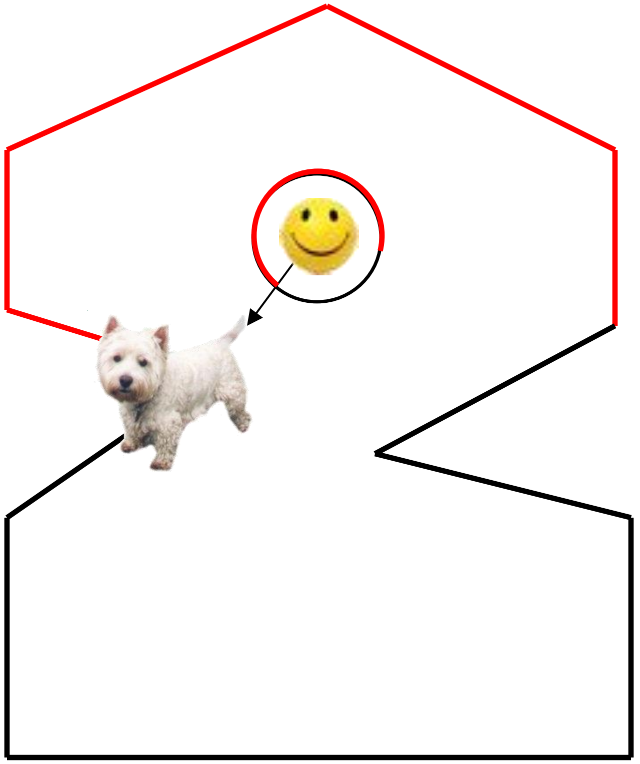

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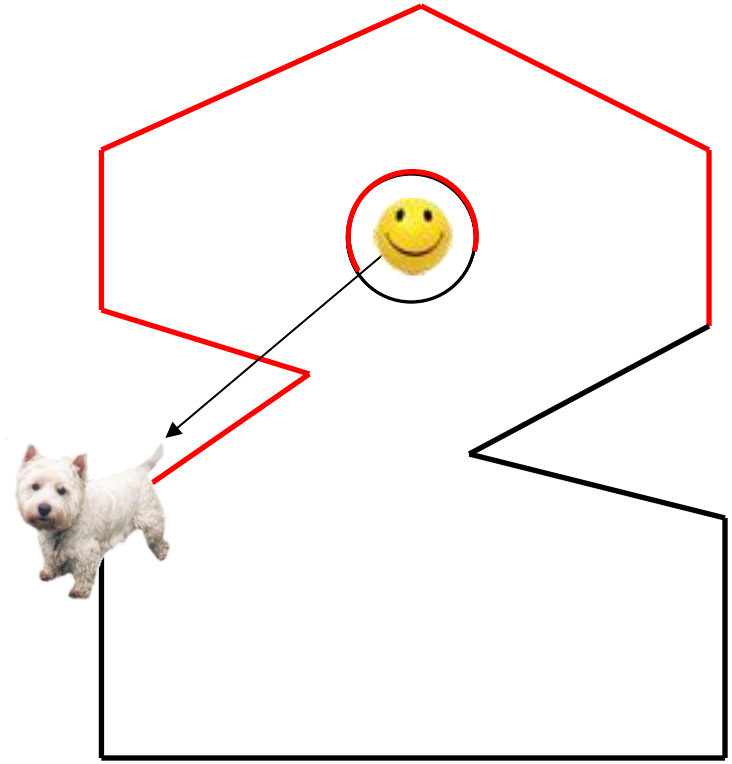

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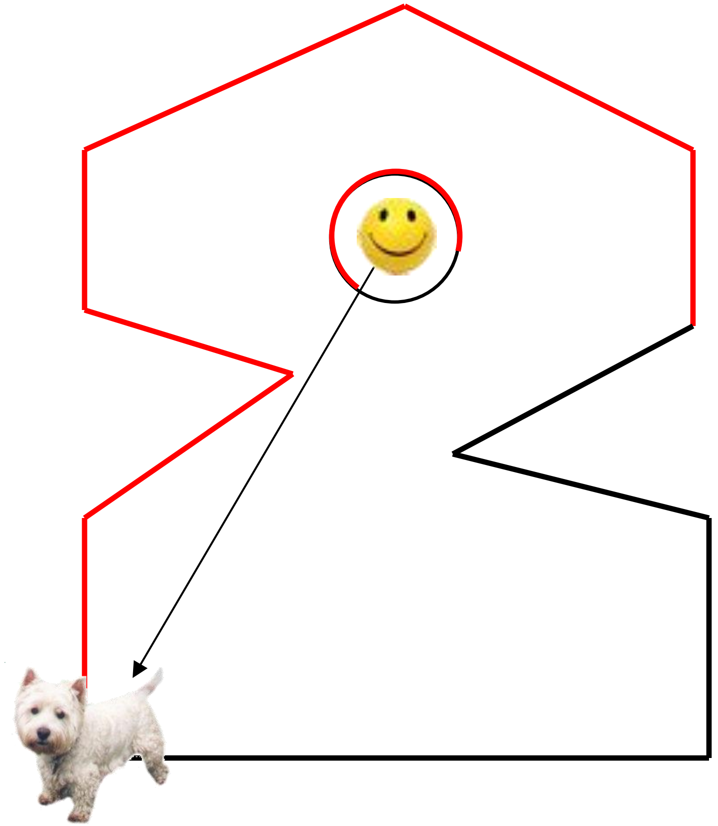

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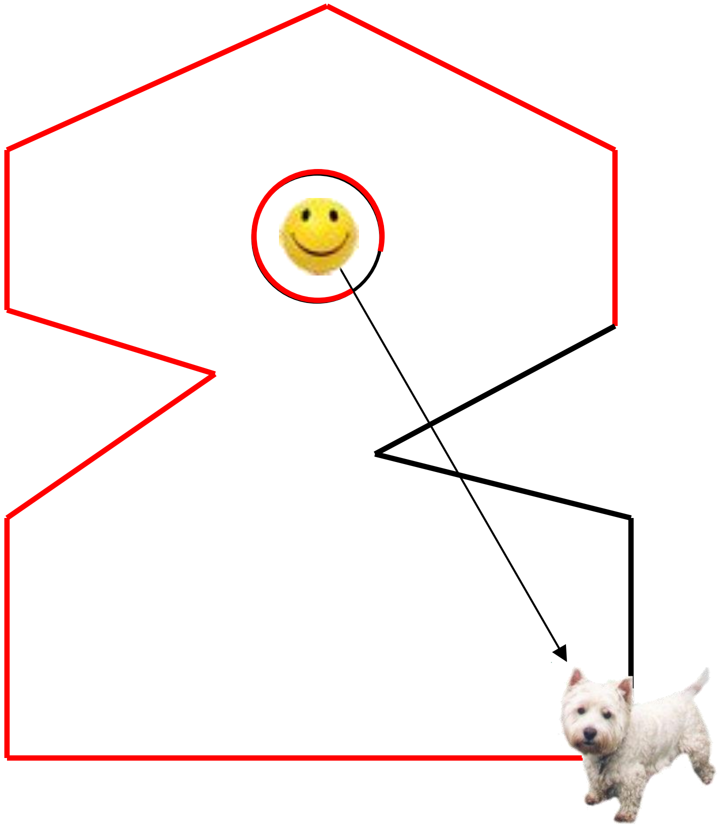

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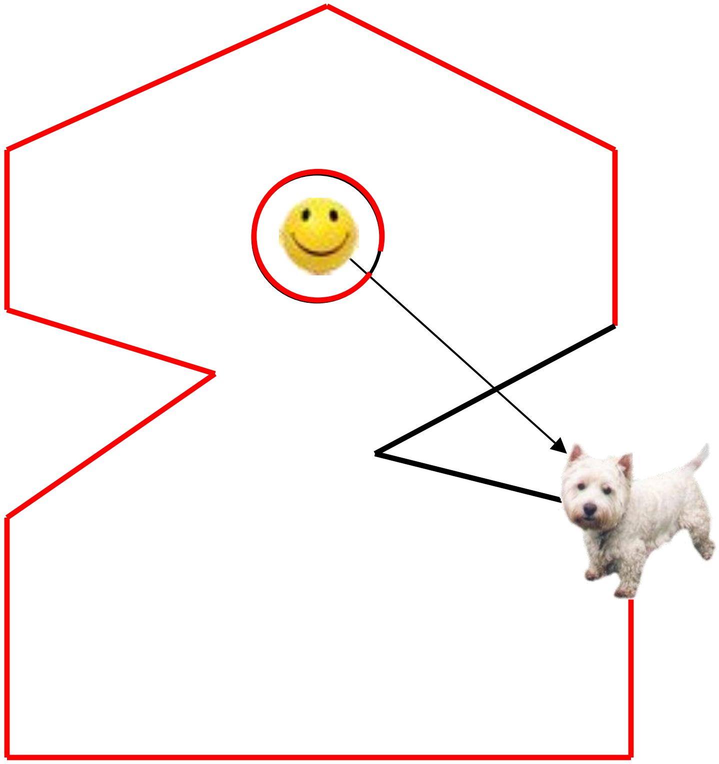

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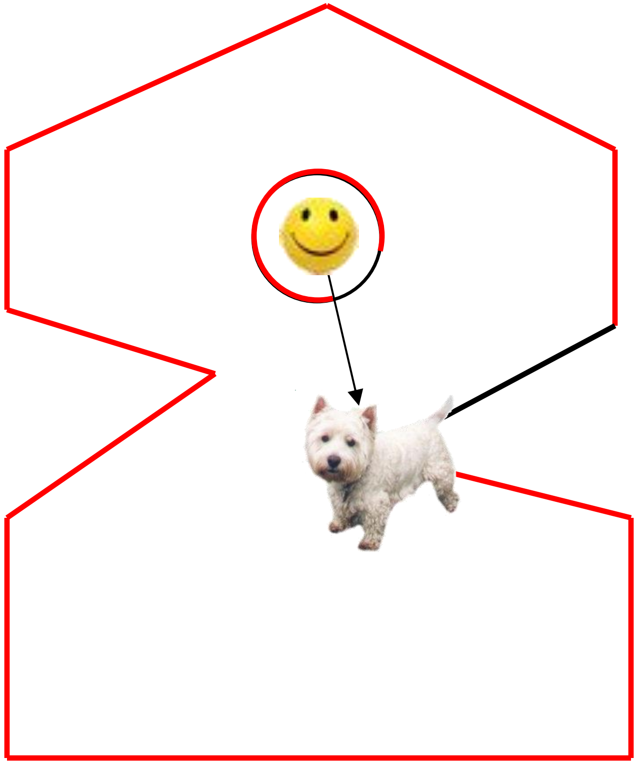
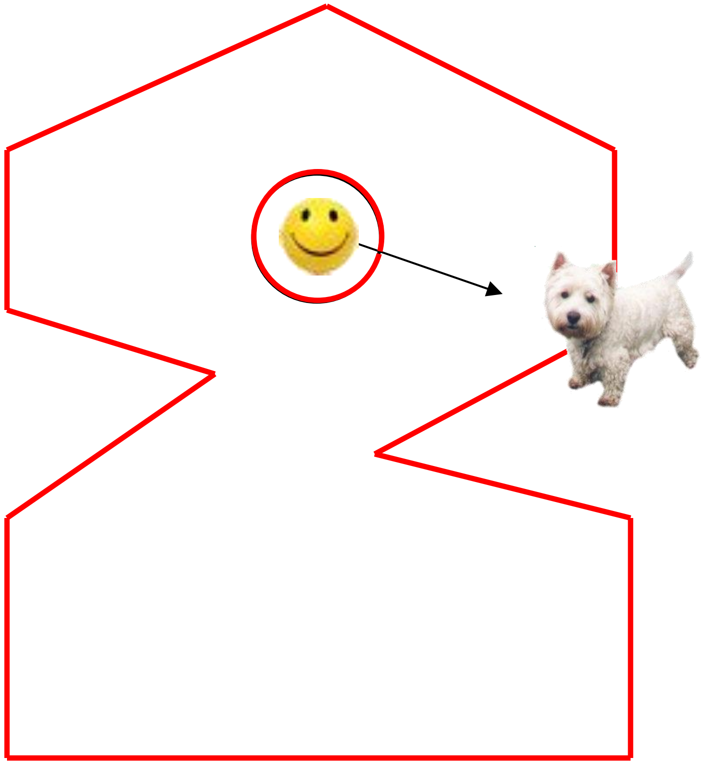


A Better Way

- One winding = inside



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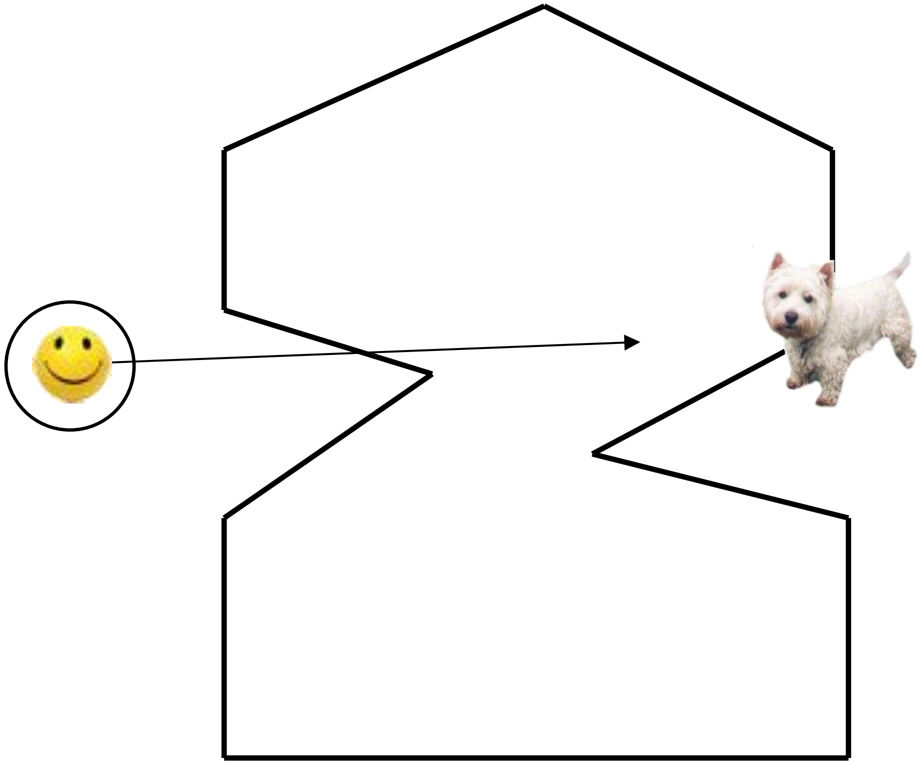

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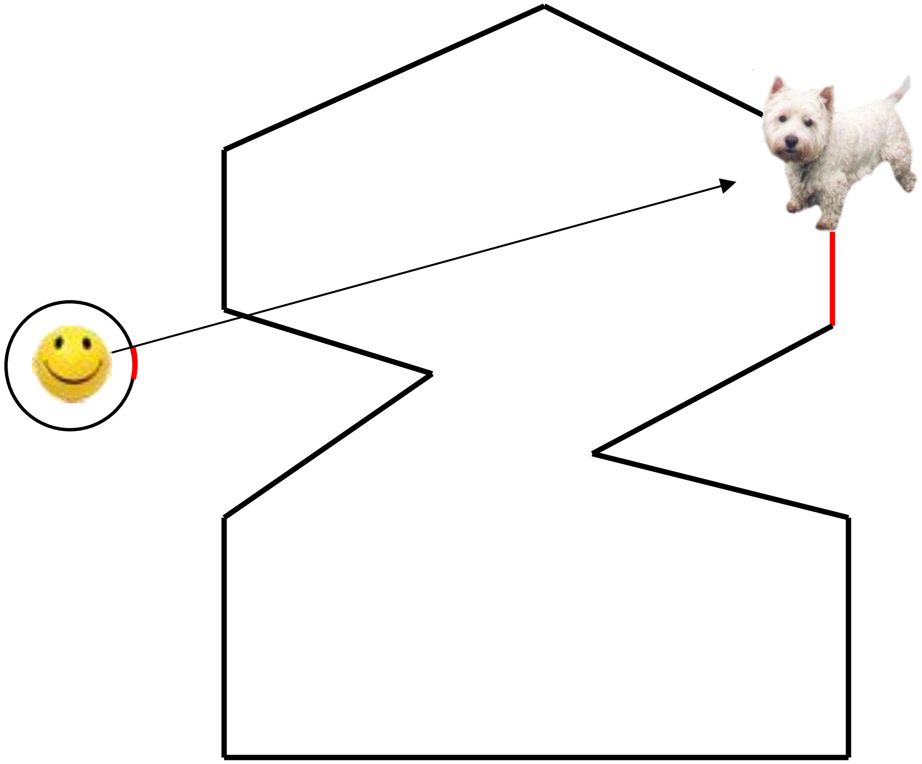

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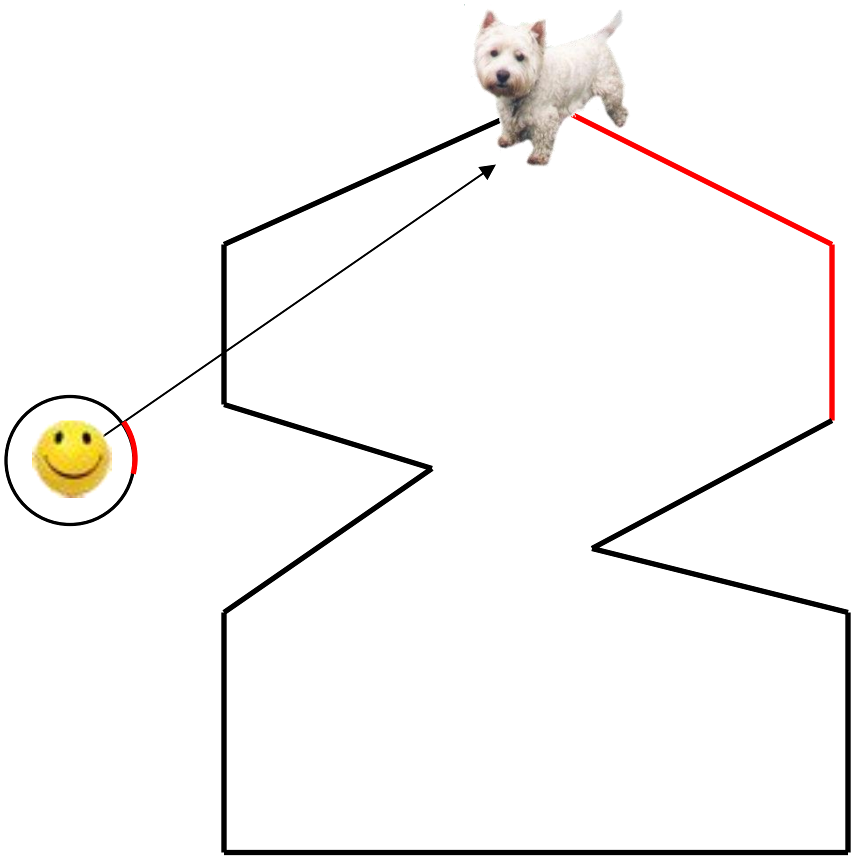

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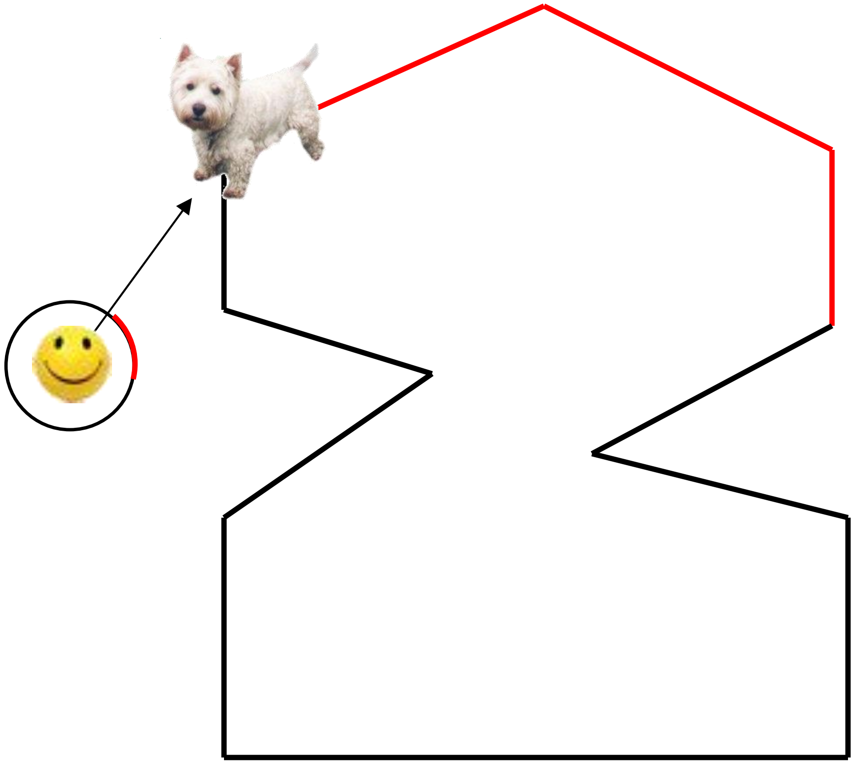

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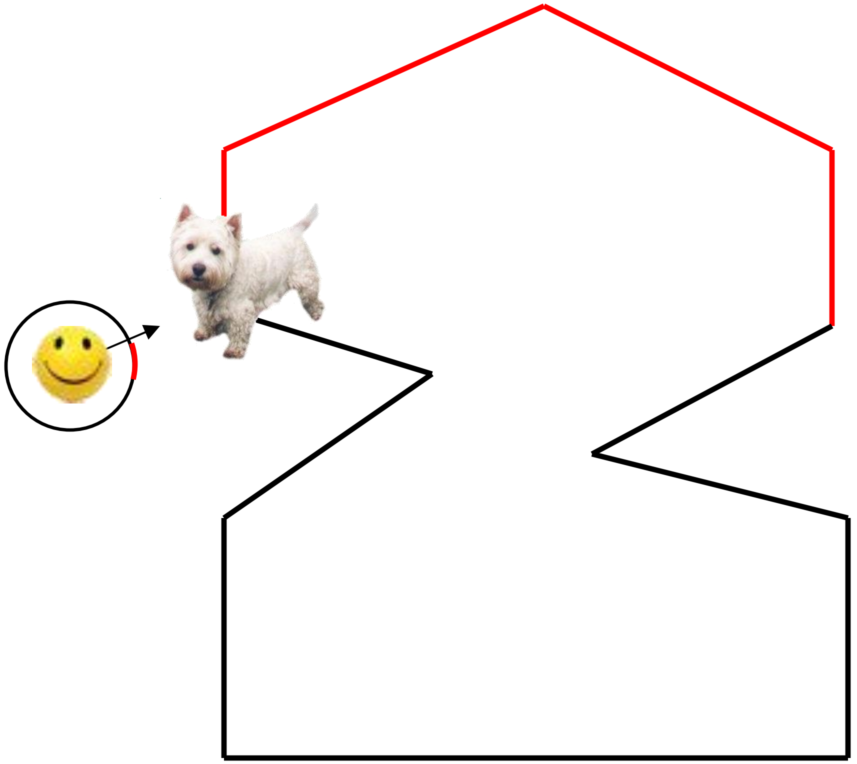

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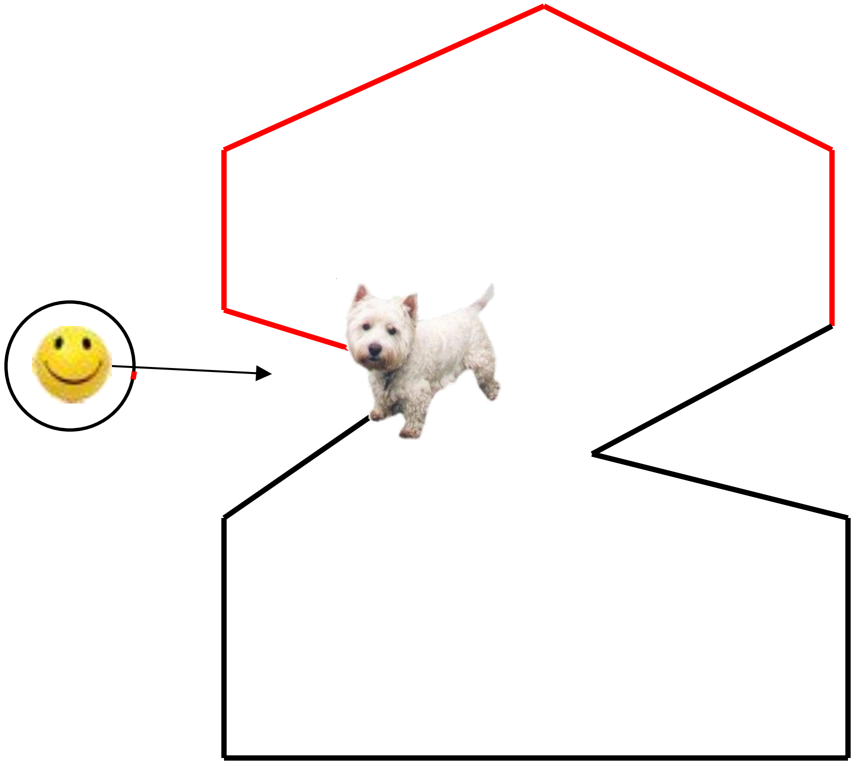

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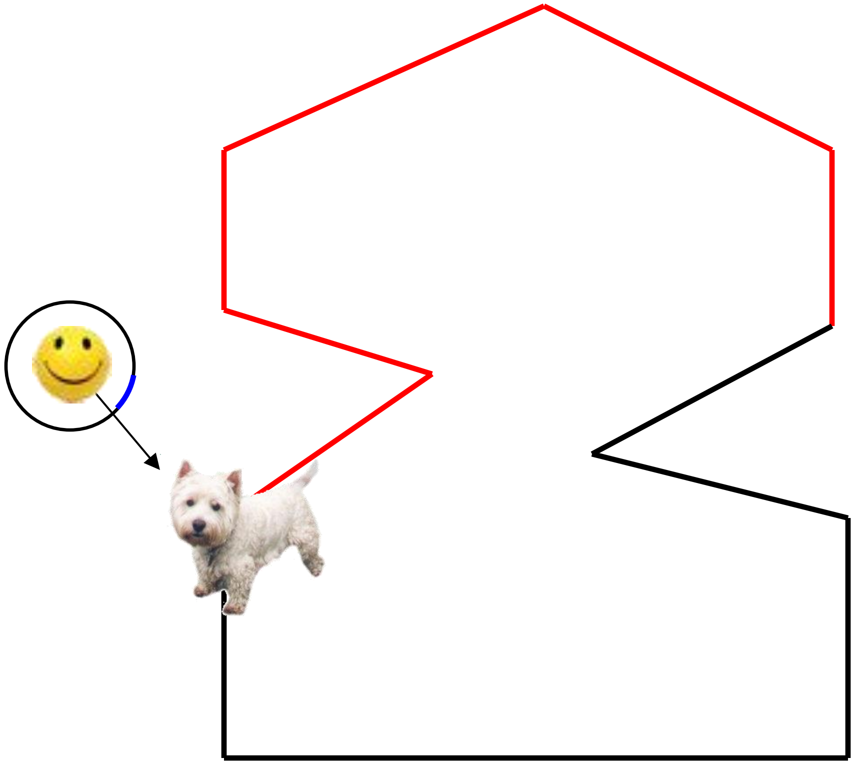

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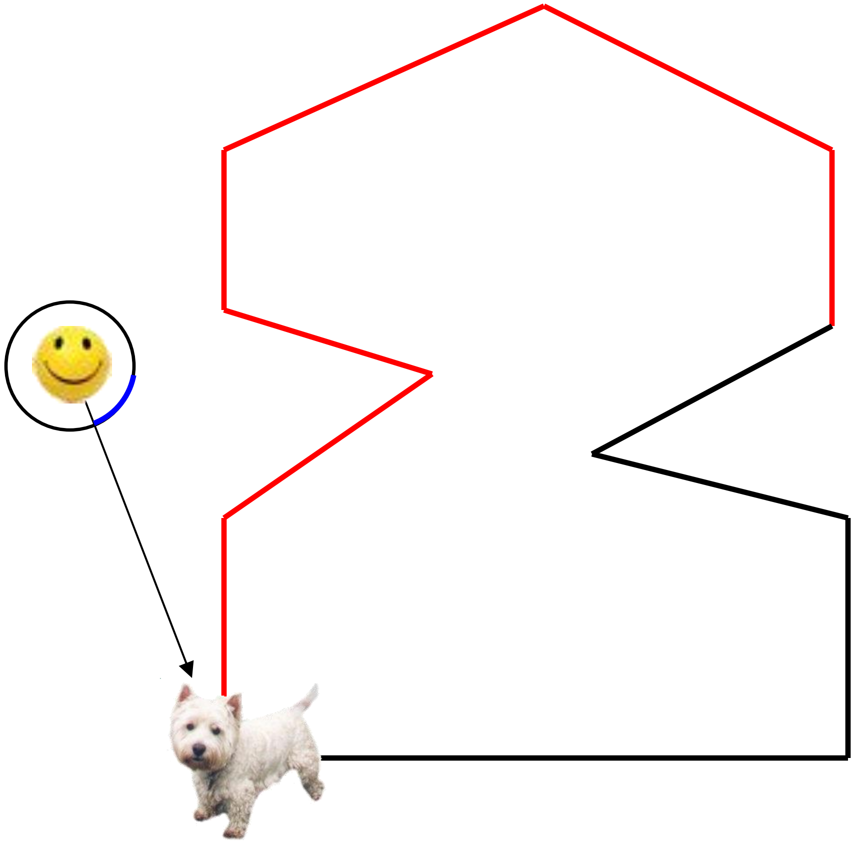

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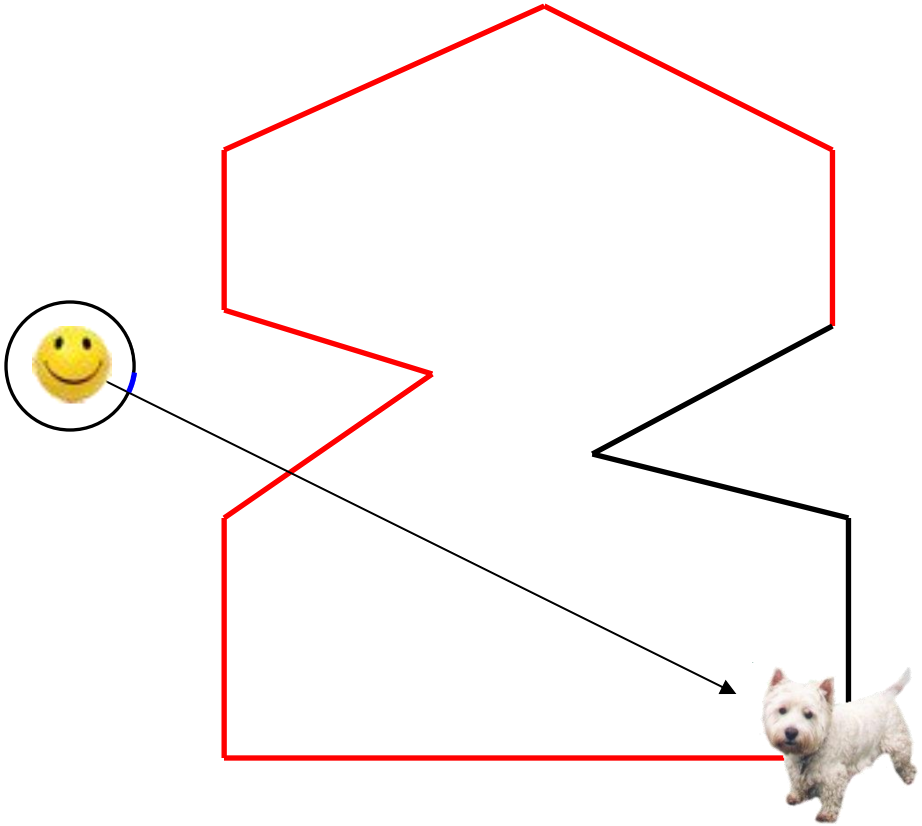

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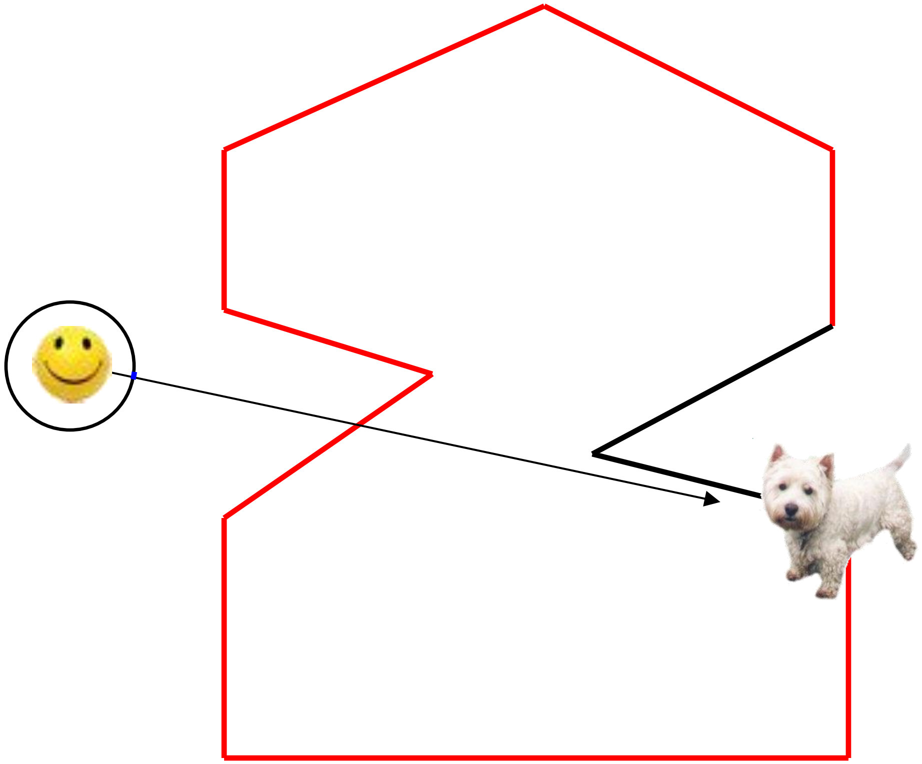

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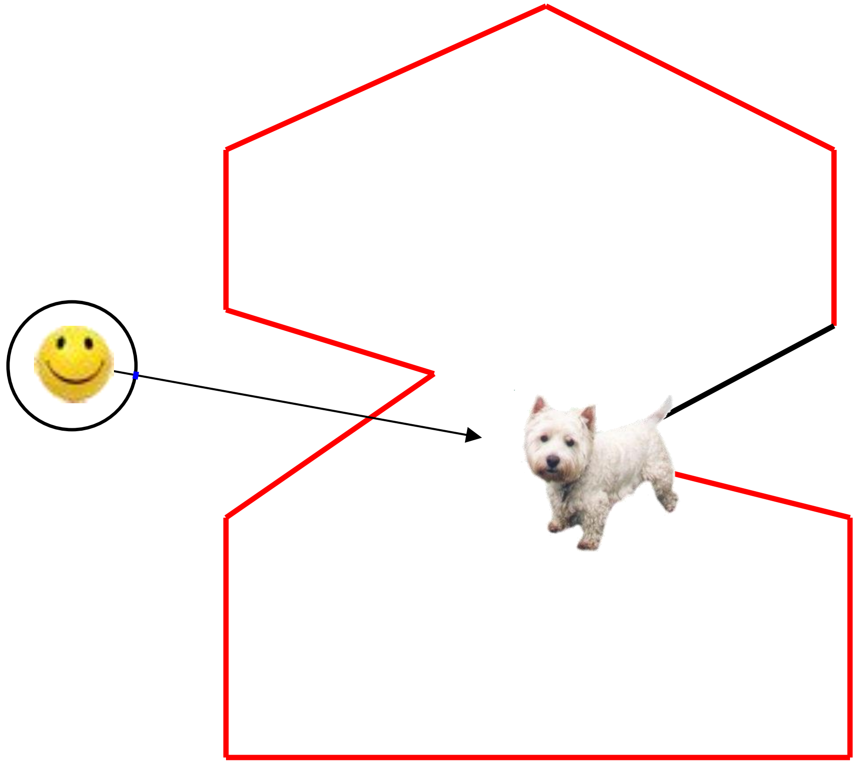
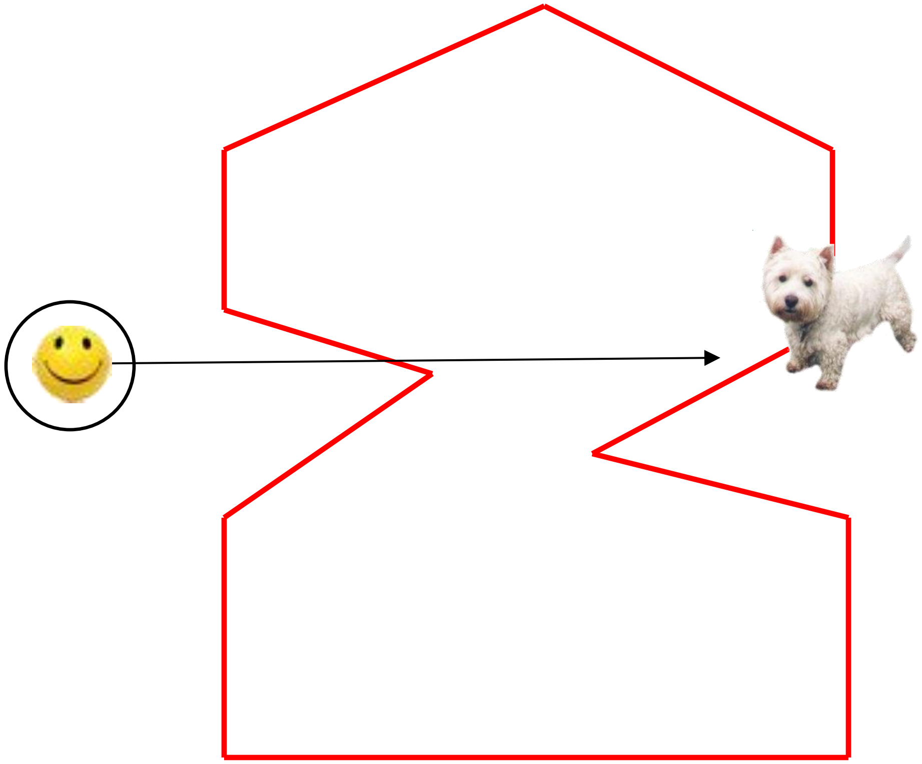


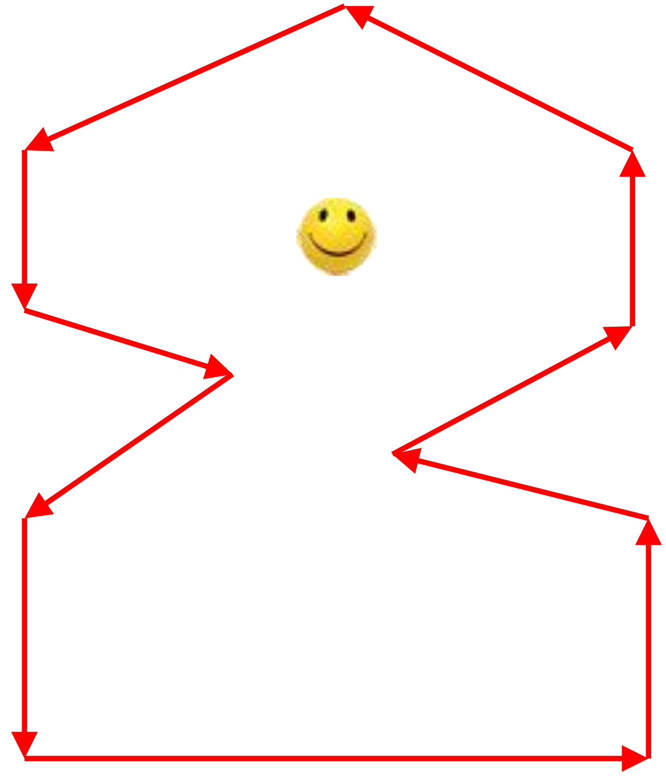
A Better Way

- zero winding = outside



Requirements

- Oriented edges
- Edges can be processed in any order

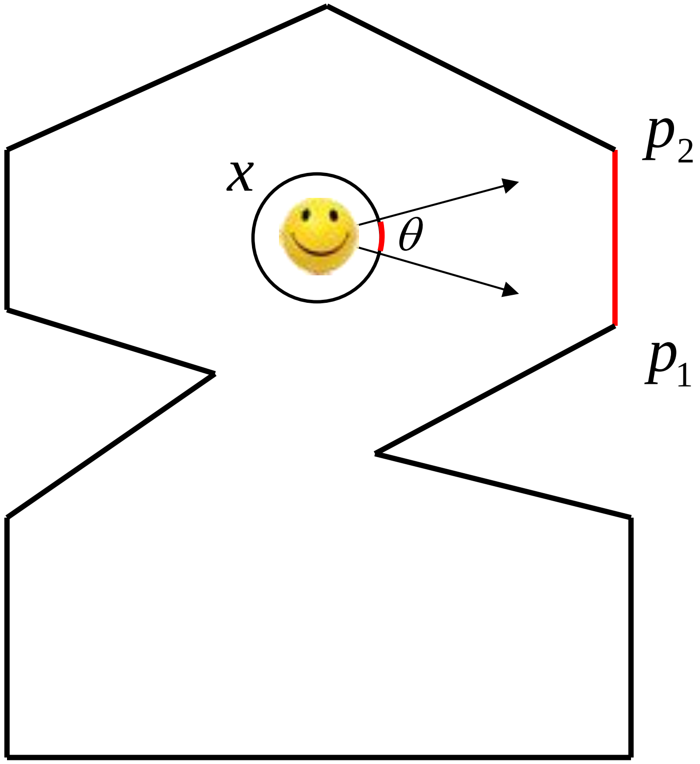


Computing Winding Number

- Given unit normal n
- $\theta = 0$
- For each edge (p_1, p_2)

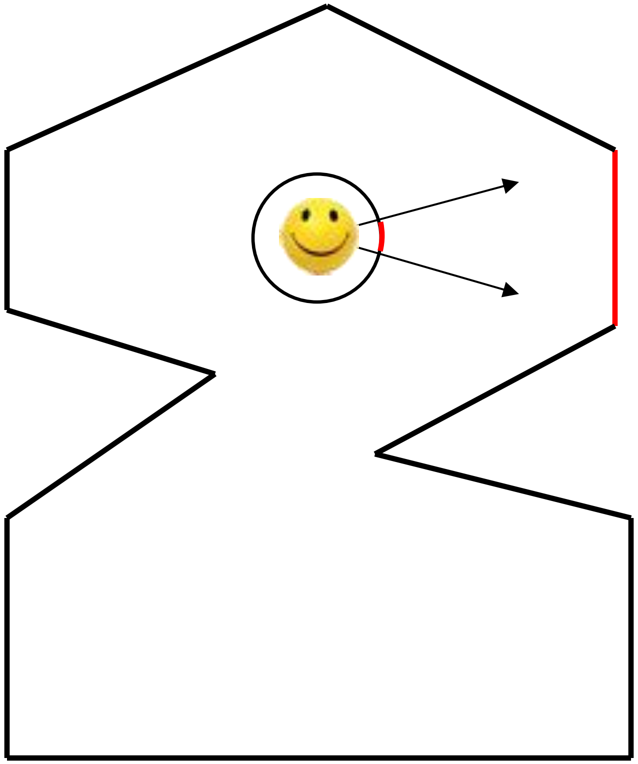
$$\theta += \frac{n \cdot ((p_1 - x) \times (p_2 - x))}{\left|(p_1 - x) \times (p_2 - x)\right|} \cos^{-1}\left(\frac{(p_1 - x) \cdot (p_2 - x)}{\left|p_1 - x\right| \left|p_2 - x\right|} \right)$$

- If $\left|\theta\right| > \pi$, then inside



Advantages

- Extends to 3D!
- Numerically stable
- Even works on models with holes (sort of)
- No ray casting



Intersecting Spheres

- Three possible cases
  - Zero intersections: miss the sphere
  - One intersection: hit tangent to sphere
  - Two intersections: hit sphere on front and back side

- How do we distinguish these cases?

Intersecting Spheres

$$F(x) = (x - c) \cdot (x - c) - r^2 = 0$$

Intersecting Spheres

$$F(x) = (x - c)\cdot(x - c) - r^2 = 0$$

$$F(L(t)) = (p + vt - c)\cdot(p + vt - c) - r^2 = 0$$

Intersecting Spheres

$$F(x) = (x - c) \cdot (x - c) - r^2 = 0$$

$$F(L(t)) = (p + vt - c) \cdot (p + vt - c) - r^2 = 0$$

$$F(L(t)) = (v \cdot v)t^2 + 2v \cdot (p - c)t + (p - c) \cdot (p - c) - r^2 = 0$$

Intersecting Spheres

- $F(L(t)) = 0$ is quadratic in t

$$F(L(t)) = \underbrace{(v \cdot v)}_{a} t^2 + \underbrace{2v \cdot (p - c)}_{b} t + \underbrace{(p - c) \cdot (p - c) - r^2}_{c} = 0$$

Intersecting Spheres

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$$F(L(t)) = \underbrace{(v \cdot v)}_{a} t^2 + \underbrace{2v \cdot (p-c)}_{b} t + \underbrace{(p-c)\cdot(p-c)-r^2}_{c} = 0$$

- Solve for t using quadratic equation

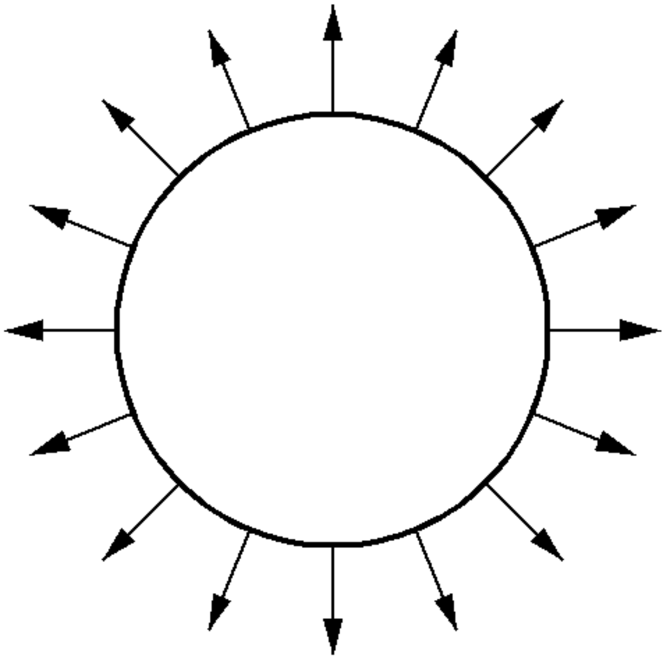
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- If $b^2-4ac<0$, no intersection
- If $b^2-4ac=0$, one intersection
- Otherwise, two intersections

Normals of Spheres

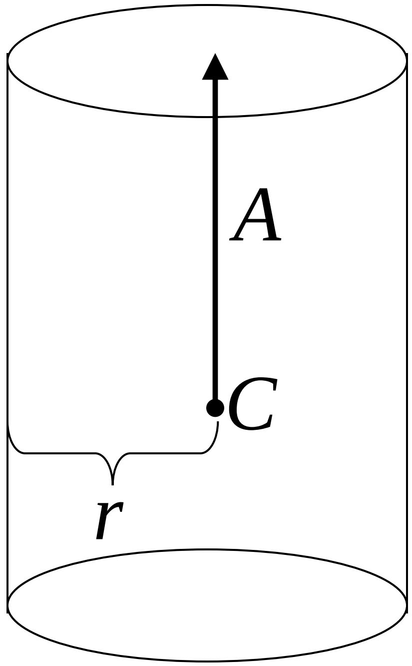
$$F(x) = (x - c) \cdot (x - c) - r^2 = 0$$

$$\nabla F(x) = x - c$$



Infinite Cylinders

- Defined by a center point *C*, a unit axis direction *A* and a radius r



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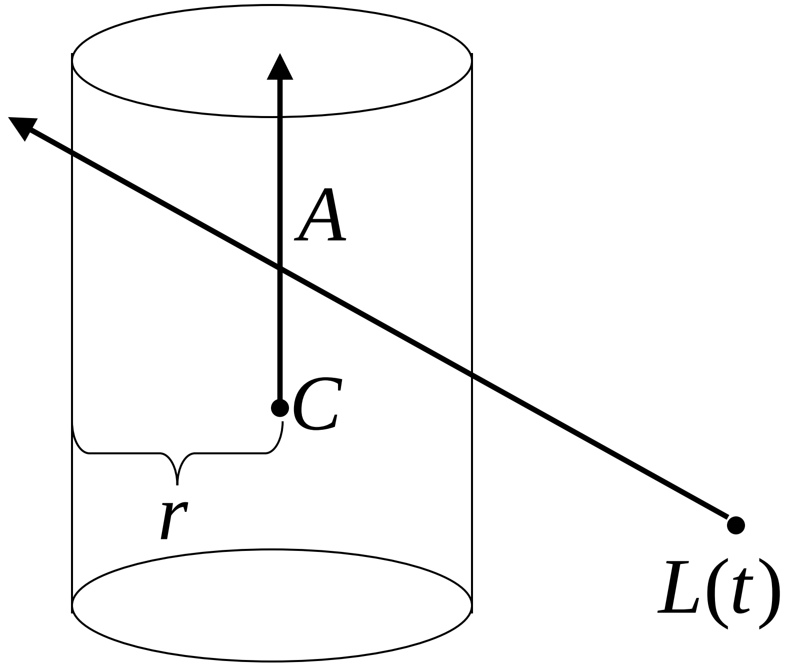
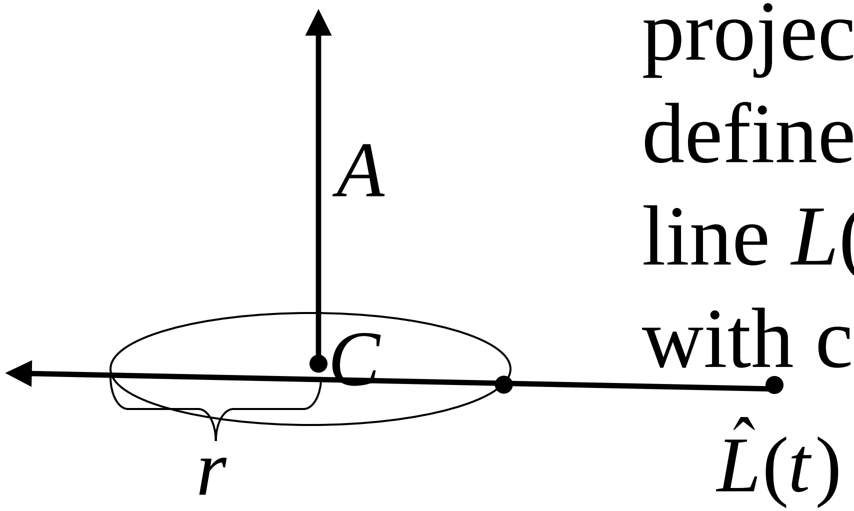


Infinite Cylinders

- Defined by a center point *C*, a unit axis direction *A* and a radius r

1.Perform an orthogonal projection to the plane defined by *C*, *A* on the line *L*(*t*) and intersect with circle in 2D



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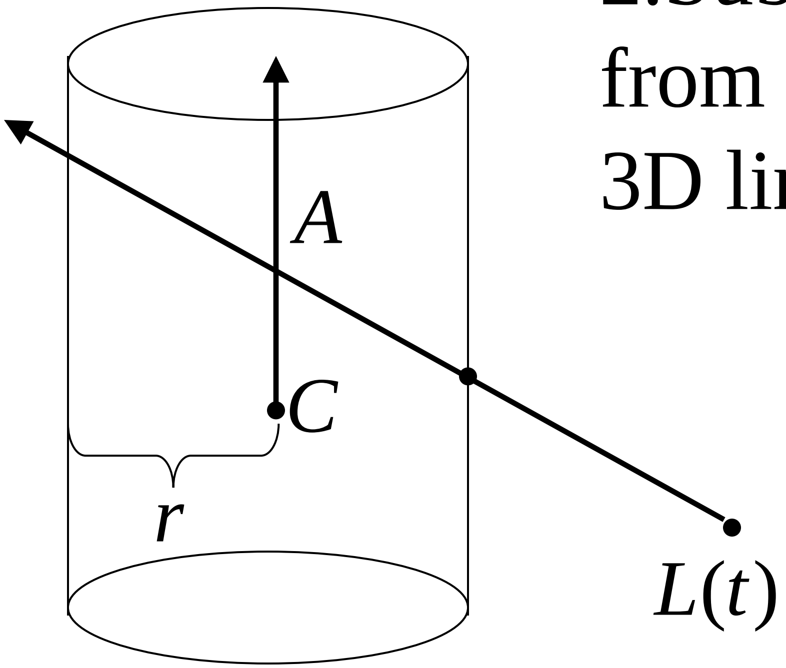
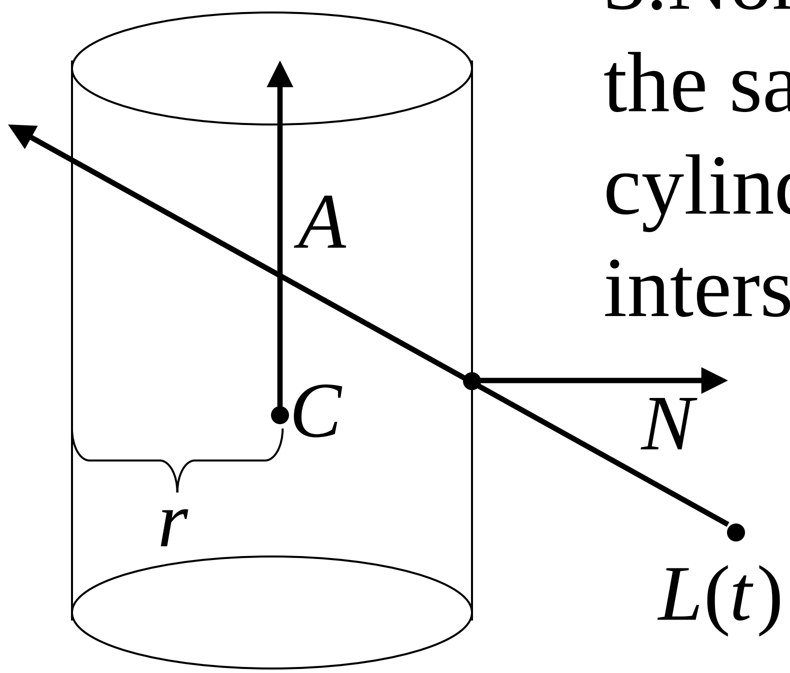


2.Substitute *t* parameters from 2D intersection to 3D line equation

Infinite Cylinders

- Defined by a center point C, a unit axis direction A and a radius r

3.Normal of 2D circle is the same normal of cylinder at point of intersection



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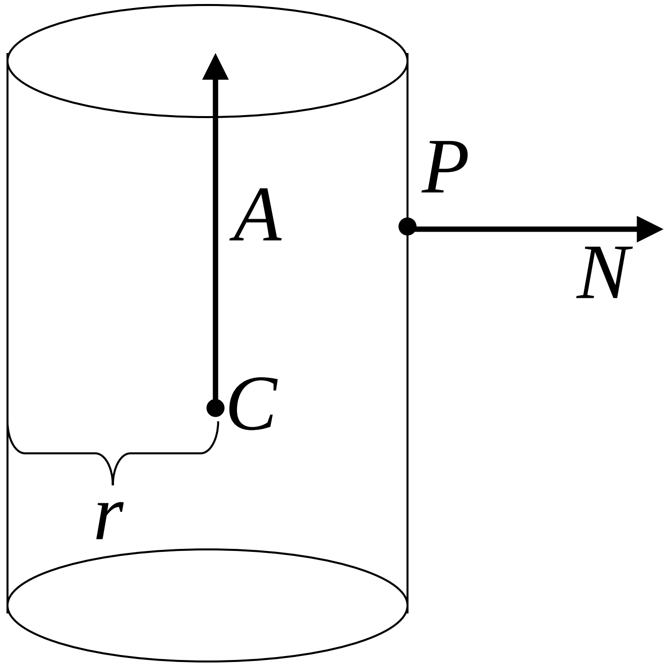

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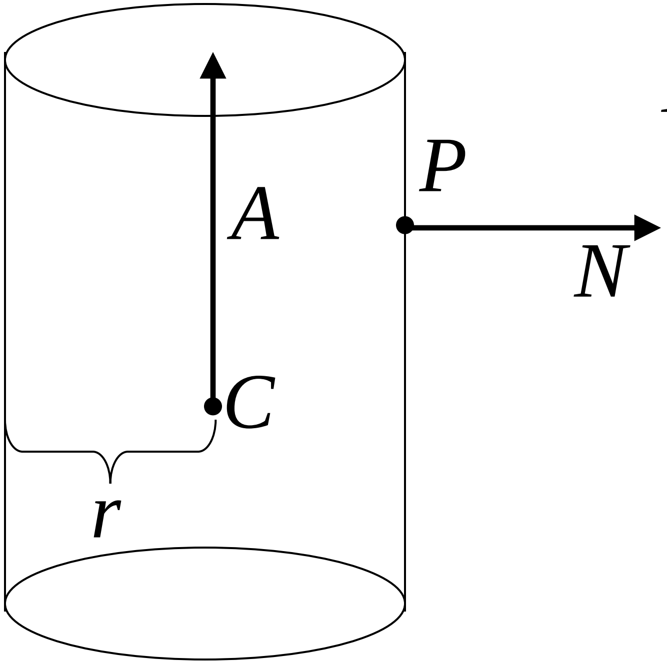

$$N = \frac{P - C - ((P - C) \cdot A)A}{r}$$