Solid Modeling

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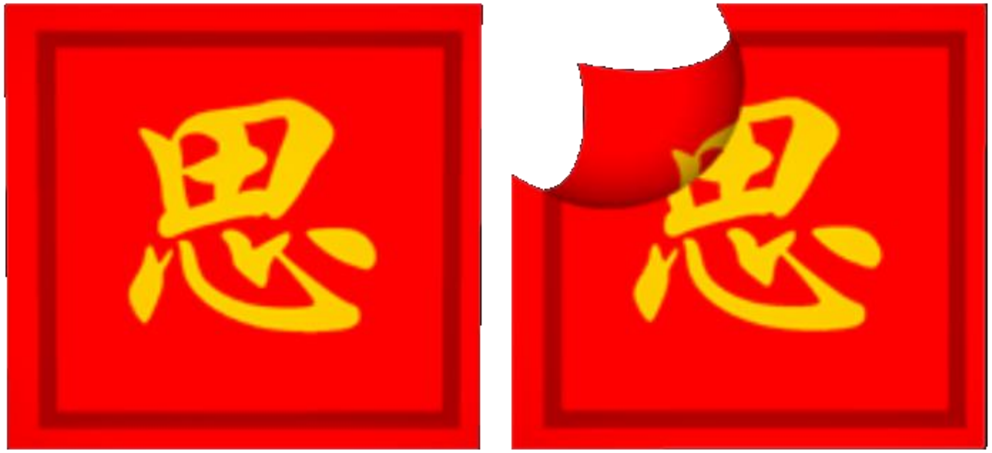
Solid Modeling Representations

- Constructive Solid Geometry
- Octrees
- Boundary Representations
- Implicit Representations

Constructive Solid Geometry

- Combine simple primitives together using set operations
  - Union, subtraction, intersection
- Intuitive operations for building more complex shapes



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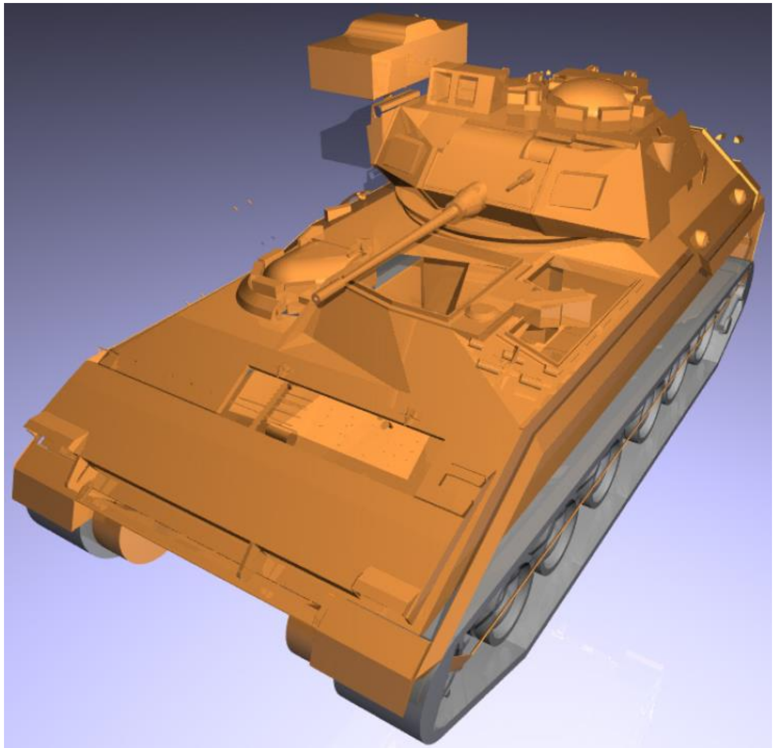
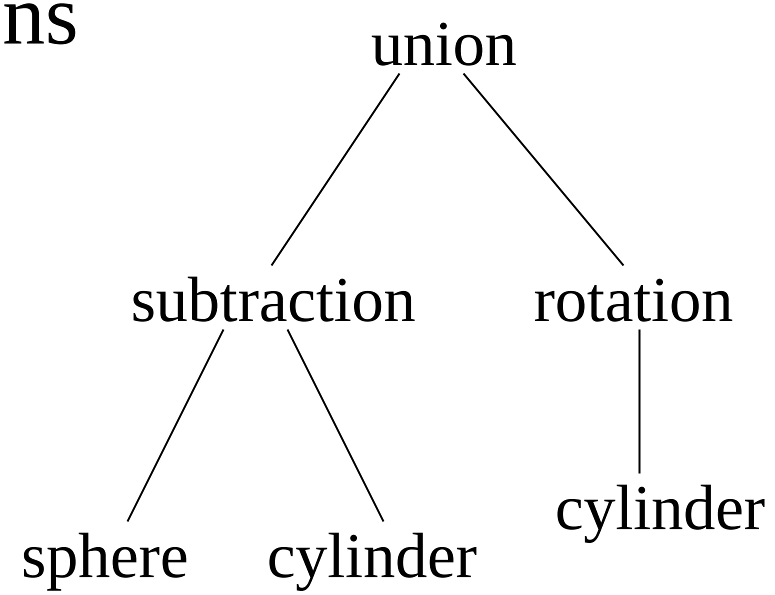


Image taken from "Feature-Sensitive Subdivision and Isosurface Reconstruction"

Constructive Solid Geometry

- Typically represented as binary tree
- Leaves store solids (sphere, cylinder, …)
- Interior nodes are operations (union, subtraction, …) or transformations



Ray Tracing CSG Trees

- Assume we have a ray R and a CSG tree T

- If T is a solid,
  - compute all intersections of R with T
  - return parameter values and normals
- If T is a transformation
  - apply inverse transformation to R and recur
  - apply inverse transpose of transformation to normals
  - return parameter values
- Otherwise T is a boolean operation
  - recur on two children to obtain two sets of intervals
  - apply operation in T to intervals
  - return parameter values.

- Display closest intersection point

Inside/Outside Test for CSG Trees

- Given a point p and a tree T, determine if p is inside/outside the solid defined by T

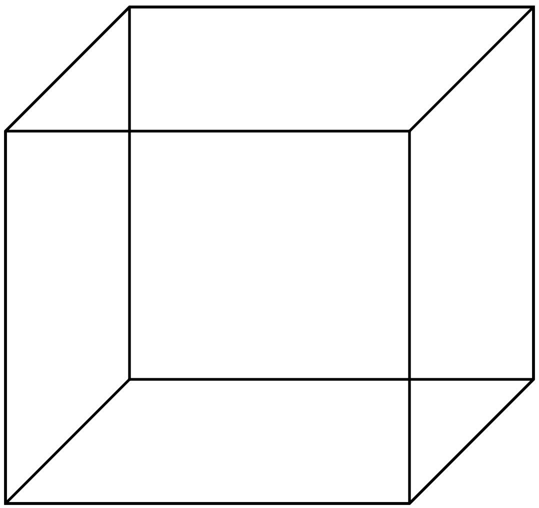
- If T is a solid
  - Determine if p is inside T and return
- If T is a transformation
  - Apply the inverse transformation to p and recur
- Otherwise T is a boolean operation
  - Recur to determine inside/outside of left/right children
  - If T is Union
    - If either child is inside, return inside, else outside
  - If T is Intersection
    - If both children are inside, return inside, else outside
  - If T is Subtraction
    - If p is inside left child and outside right child, return inside, else outside

Application: Computing Volume

- Monte Carlo method
- Put bounding box around object
- Pick n random points inside the box
  - Determine if each point is inside/outside the CSG Tree
- Volume $\approx vol(box)\,{}^{\#inside}\!/_{n}$

Octrees

- Models space as a tree with 8 children
- Nodes can be 3 types
  - Interior Nodes
  - Solid
  - Empty



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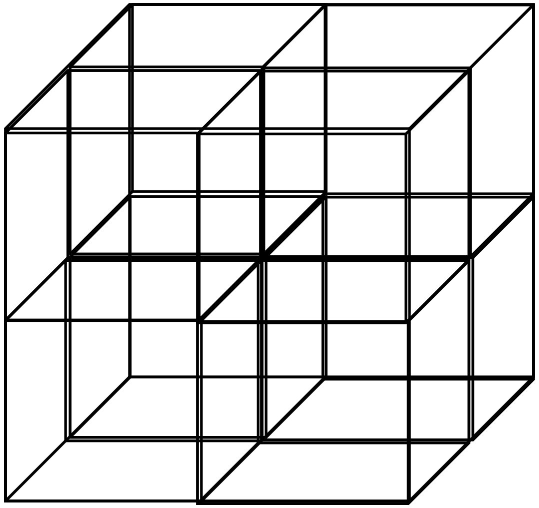
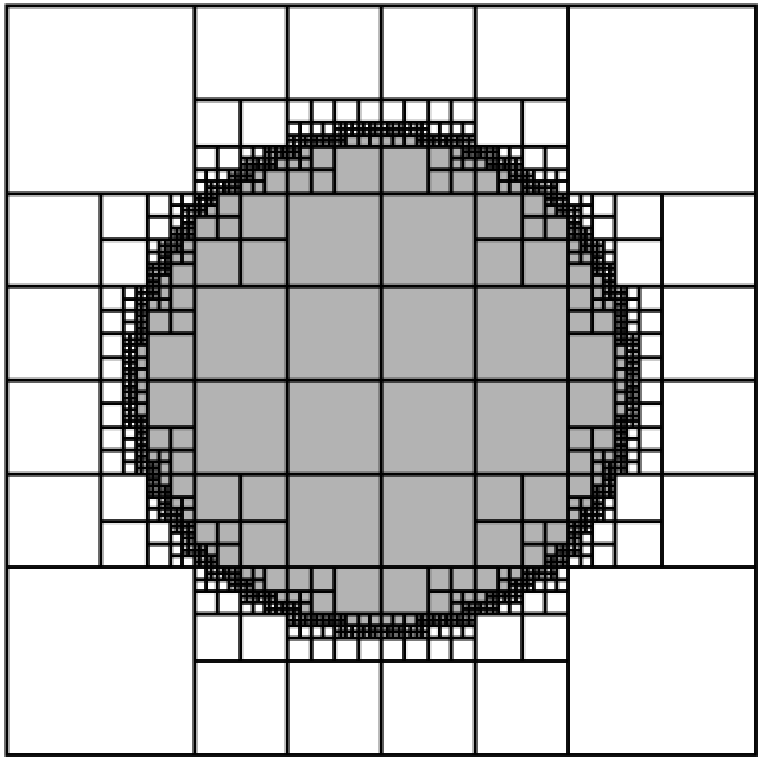


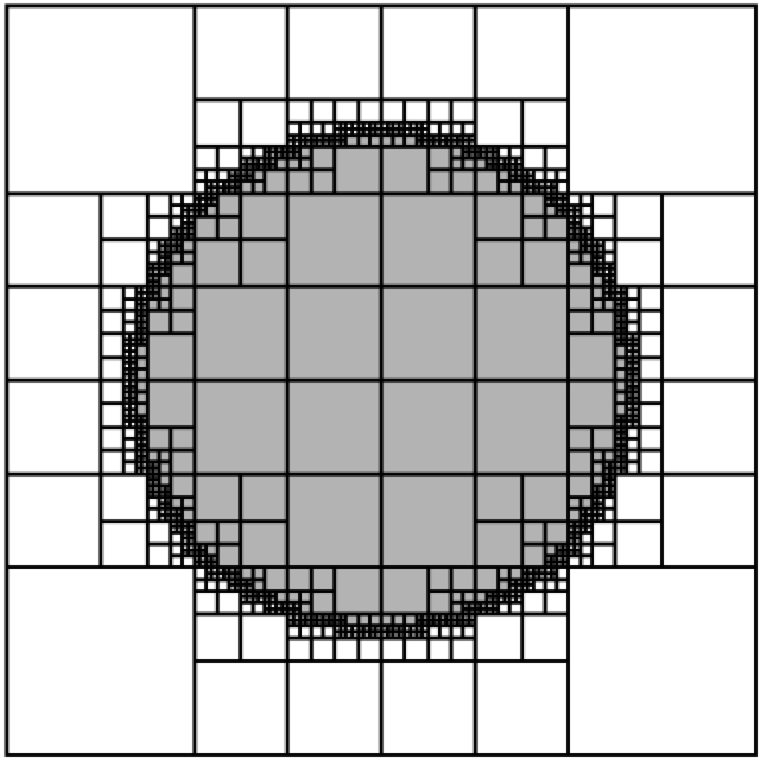
Building Octrees

- If cube completely inside, return solid node
- If cube completely outside, return empty node
- Otherwise recur until maximum depth reached



Octrees

- Advantages
  - Storage space proportional to surface area
  - Inside/Outside trivial
  - Volume trivial
  - CSG relatively simple
  - Can approximate any shape
- Disadvantages
  - Blocky appearance



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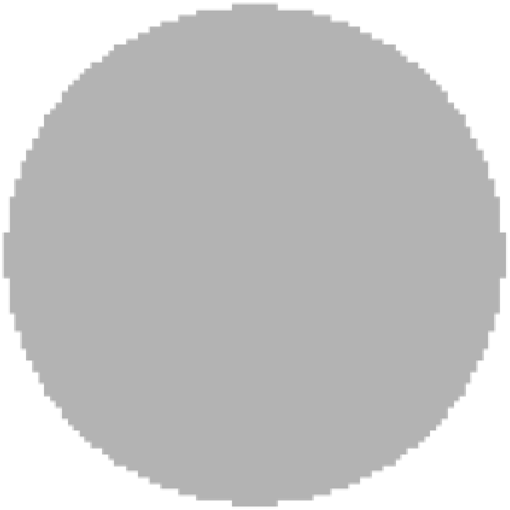
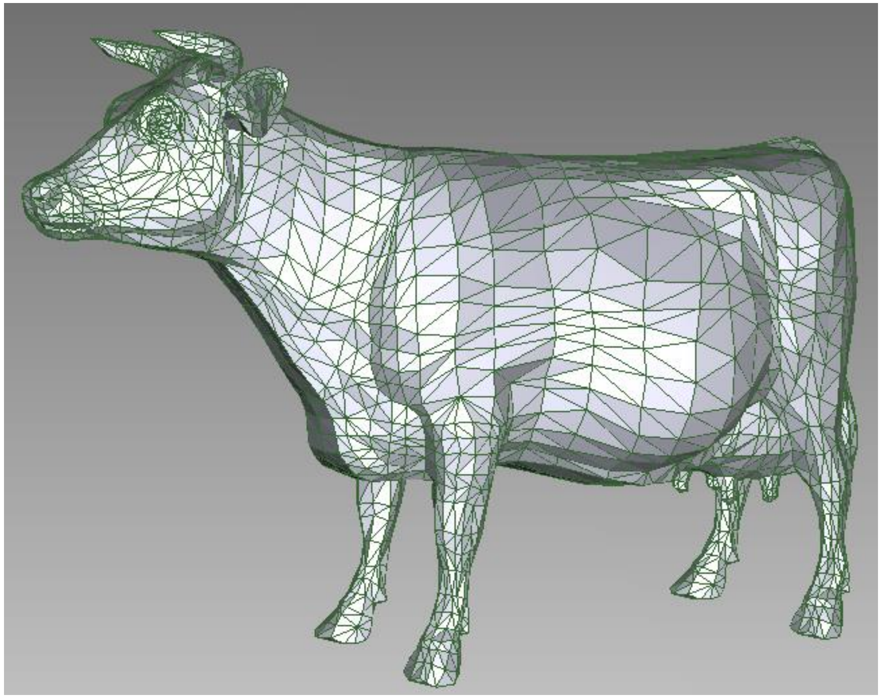


Boundary Representations

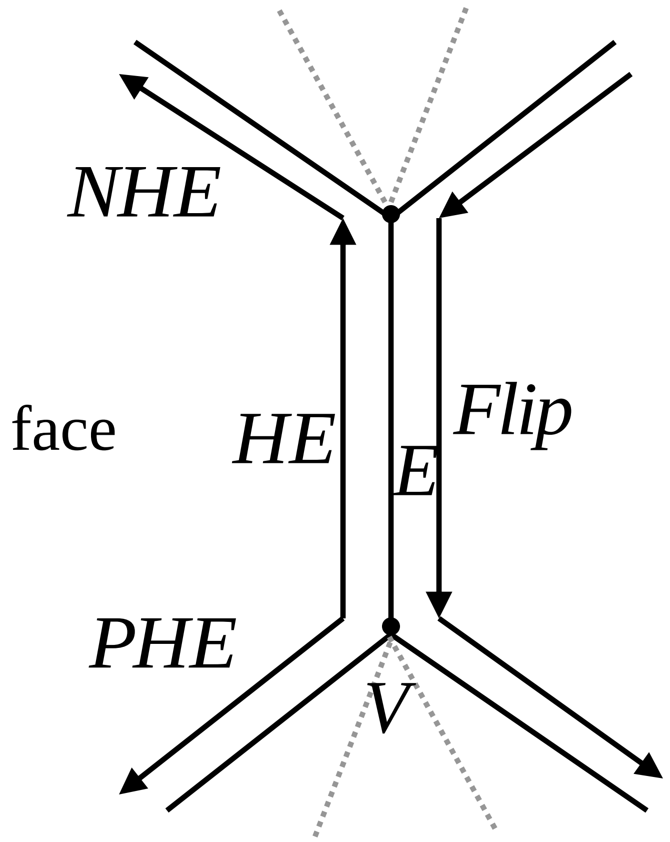
- Stores the boundary of a solid
  - Geometry: vertex locations
  - Topology: connectivity information
    - Vertices
    - Edges
    - Faces



Boundary Representations

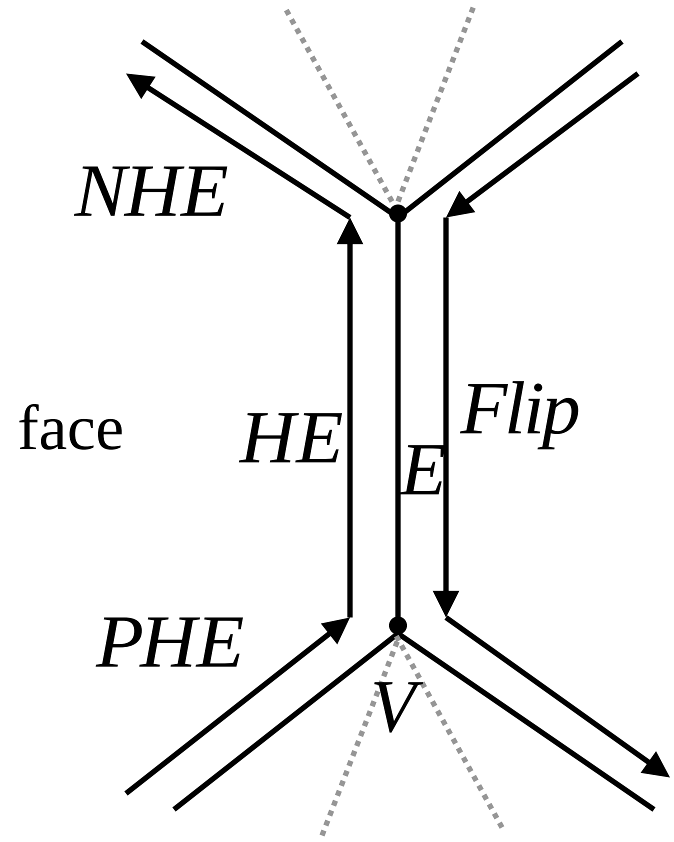
- Constant time adjacency information
  - For each vertex,
    - Find edges/faces touching vertex
  - For each edge,
    - Find vertices/faces touching edge
  - For each face,
    - Find vertices/edges touching face

Half Edge Data Structure



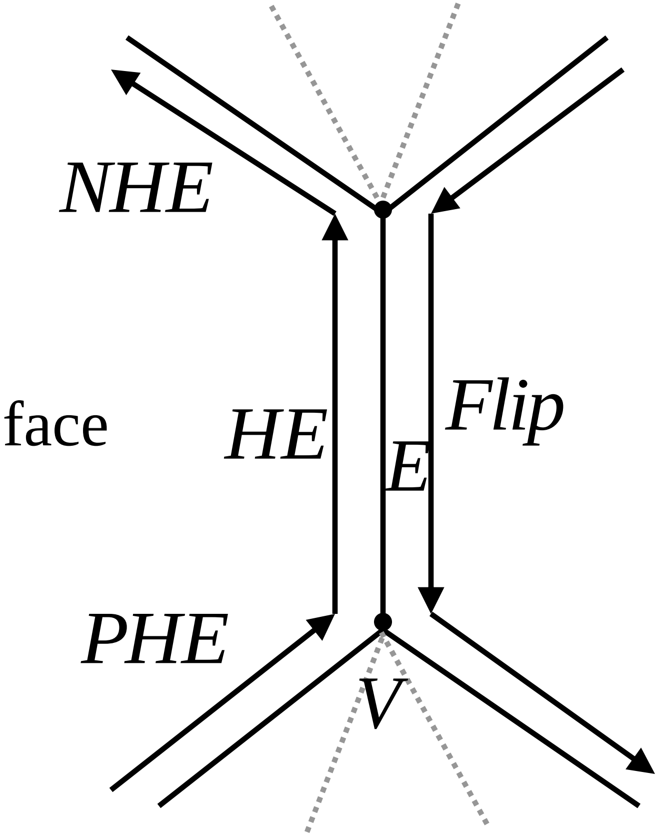
Half Edge Data Structure

```
HalfEdge {
    HalfEdge next, prev, flip;
    Face face;
    Vertex origin;
    Edge edge;
}
Face {
    HalfEdge edge; // part of this face
}
Vertex {
    HalfEdge edge;  // points away
}
Edge {
    HalfEdge he;
}
```



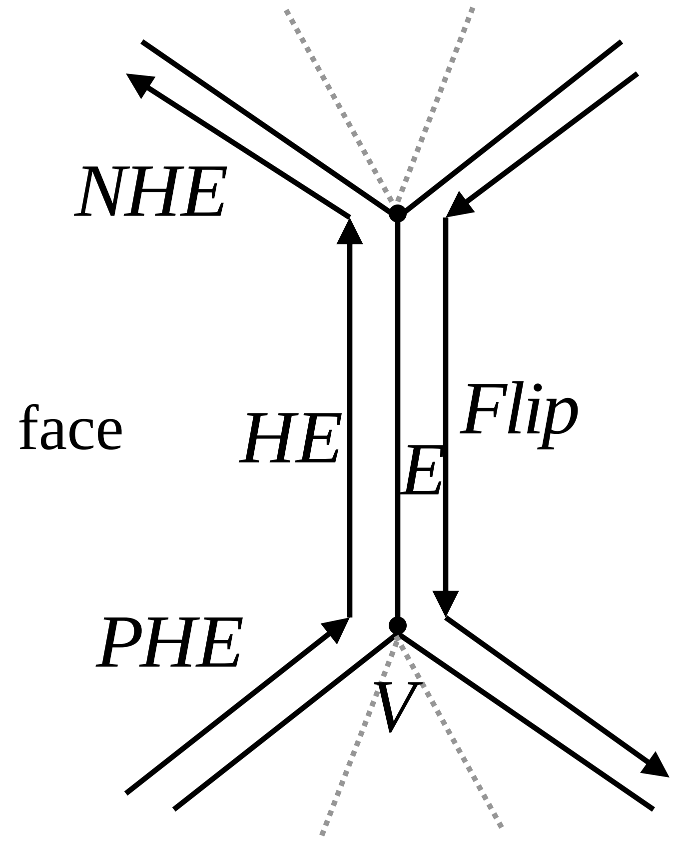
Half Edge Data Structure

- Given a face, find all vertices touching that face
- Given a vertex, find all edge-adjacent vertices
- Given a face, find all adjacent faces



Building a Topological Data Structure

- Must connect adjacent edges/faces/vertices
- Edges are critical in most data structures
- Use a hash table indexed by two vertices



Boundary Representations

- Advantages
  - Explicitly stores neighbor information
  - Easy to render
  - Easy to calculate volume
  - Nice looking surface
- Disadvantages
  - CSG very difficult
  - Inside/Outside test hard

Implicit Representations of Shape

- Shape described by solution to $f(x)=c$

$$f(x, y) = x^2 + y^2 - 9$$

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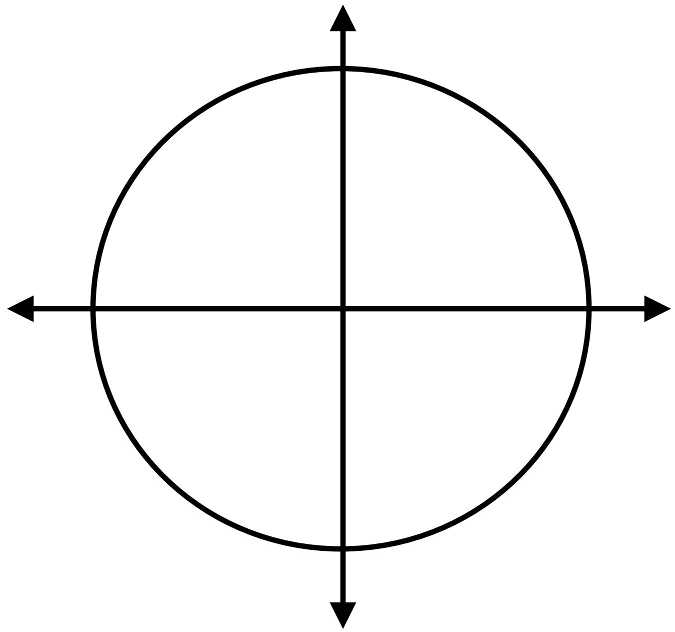

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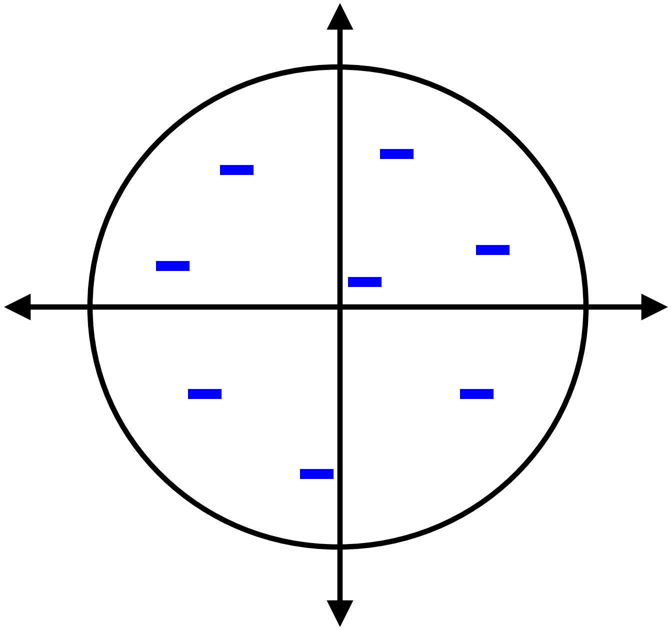

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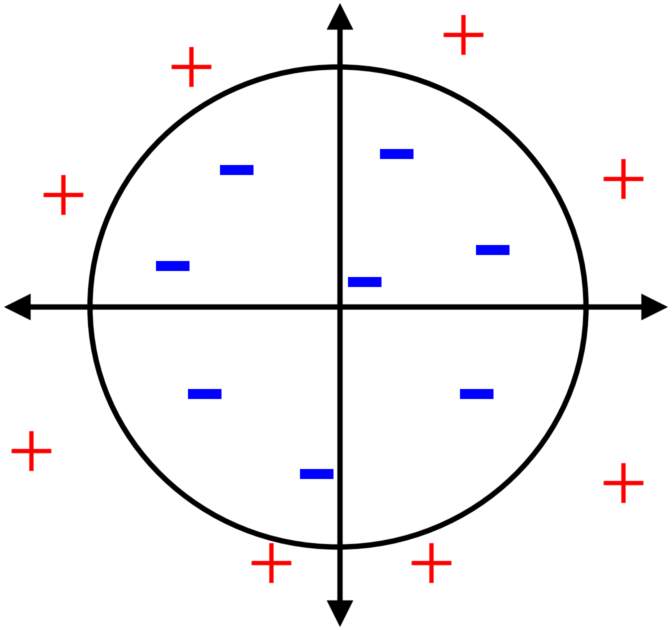

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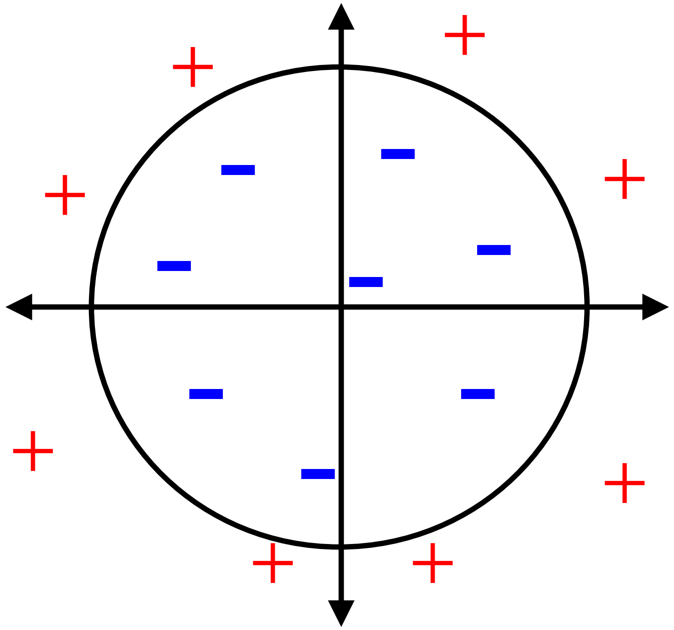

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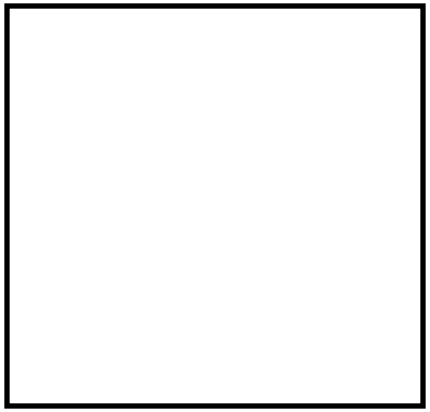

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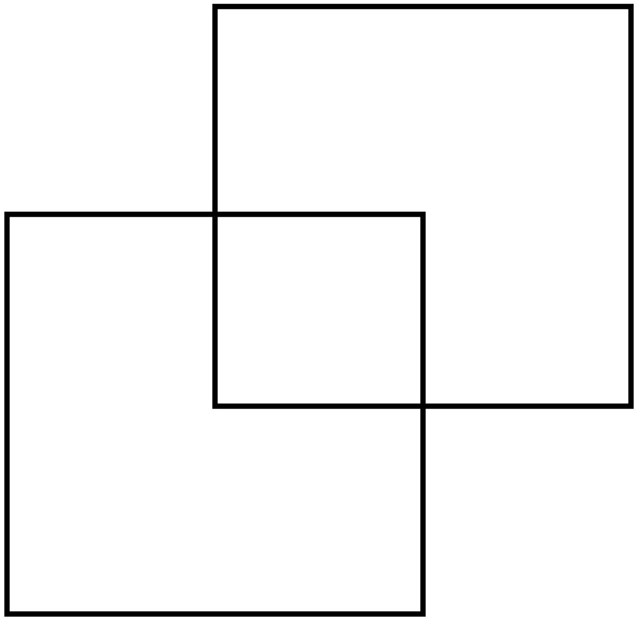

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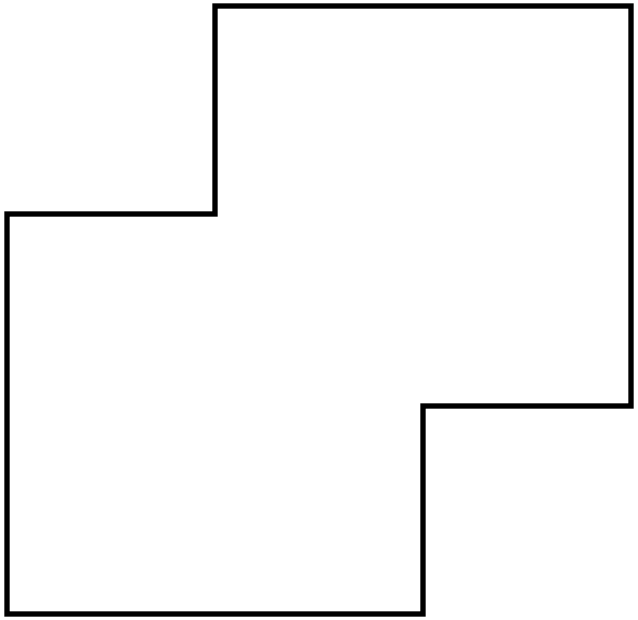

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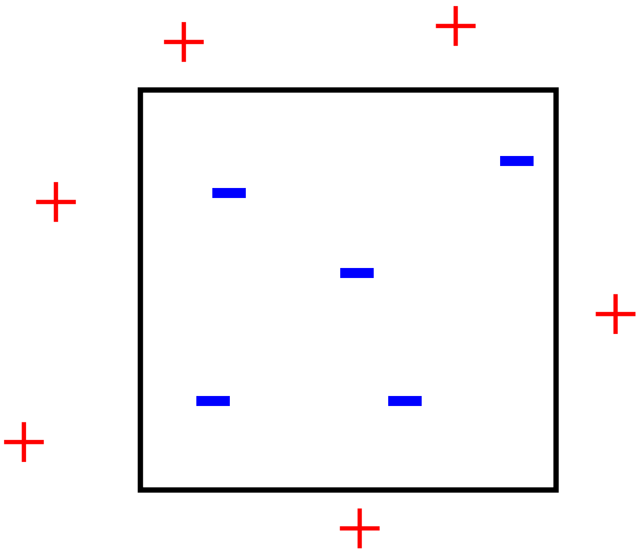

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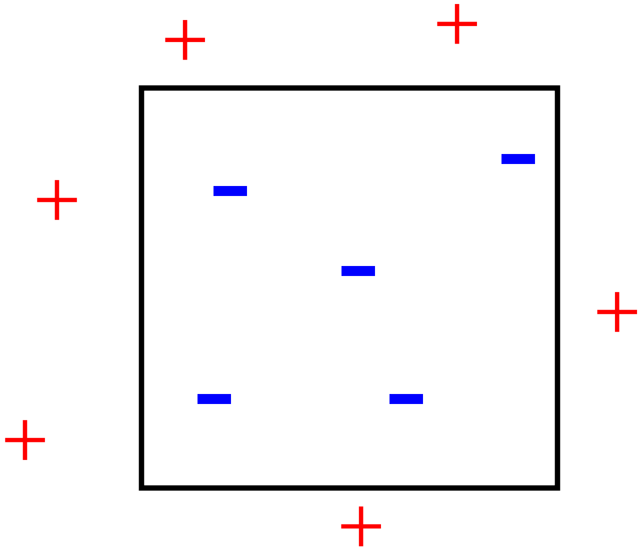

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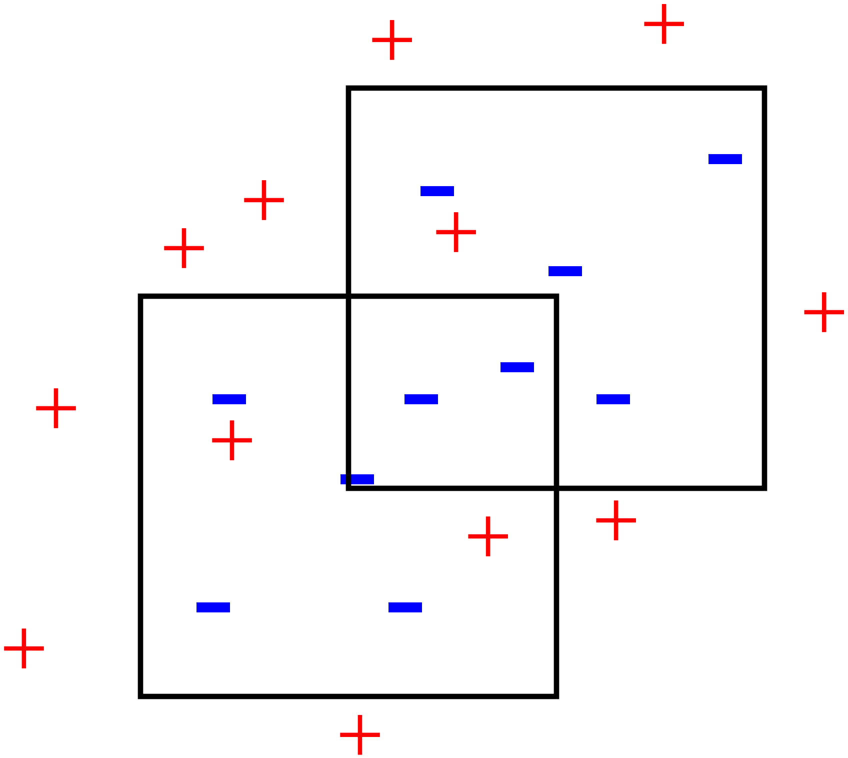

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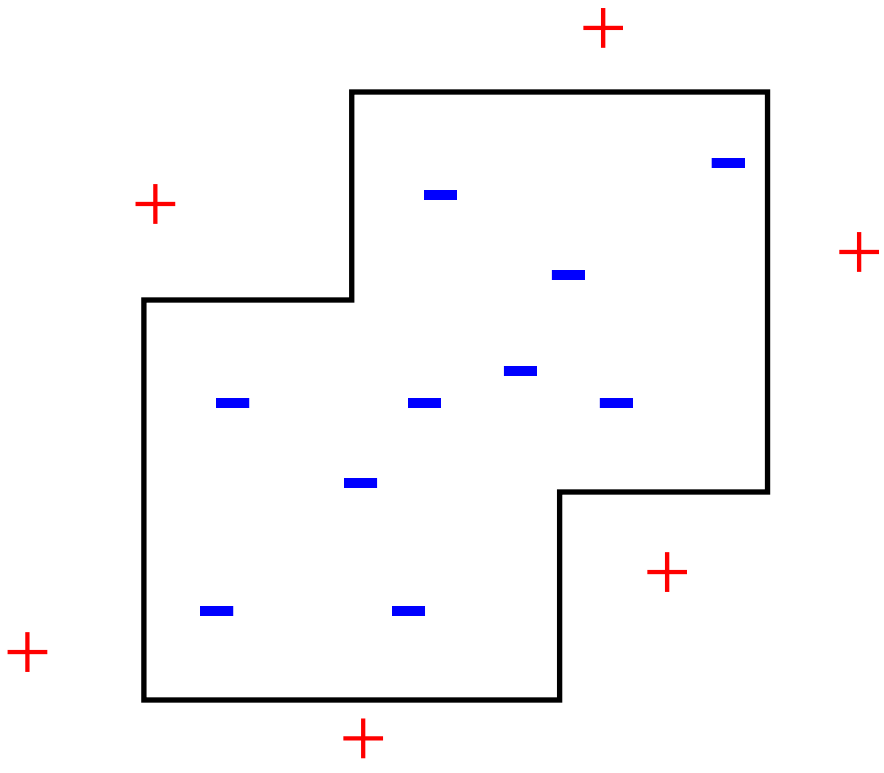

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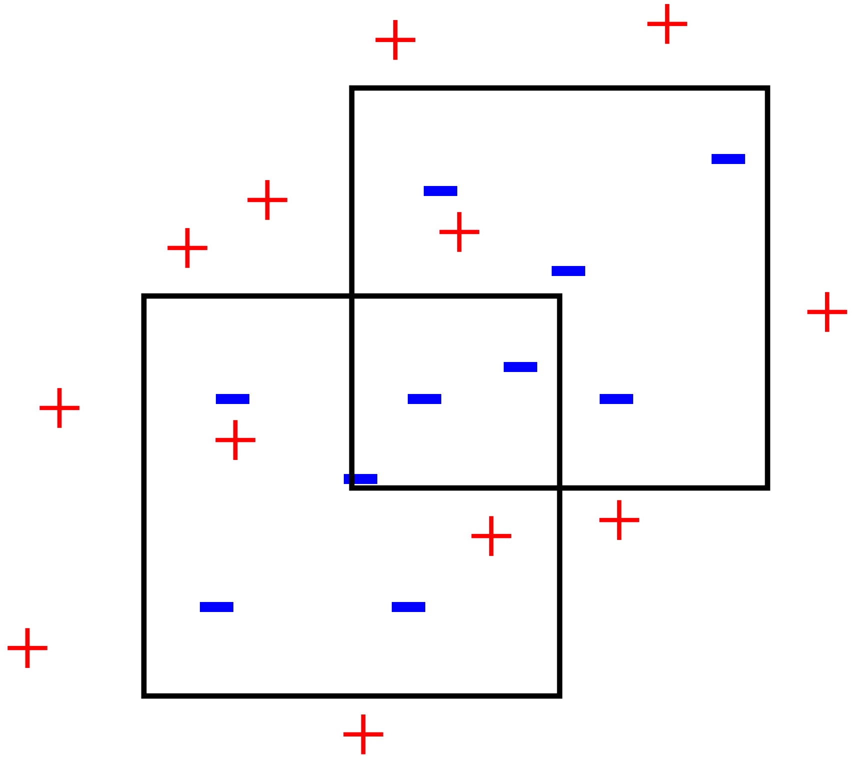

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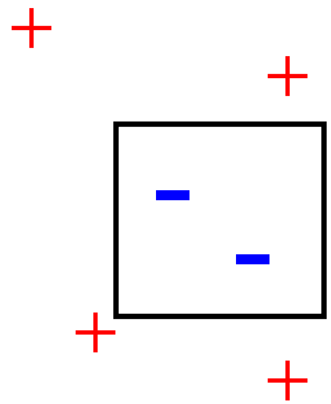

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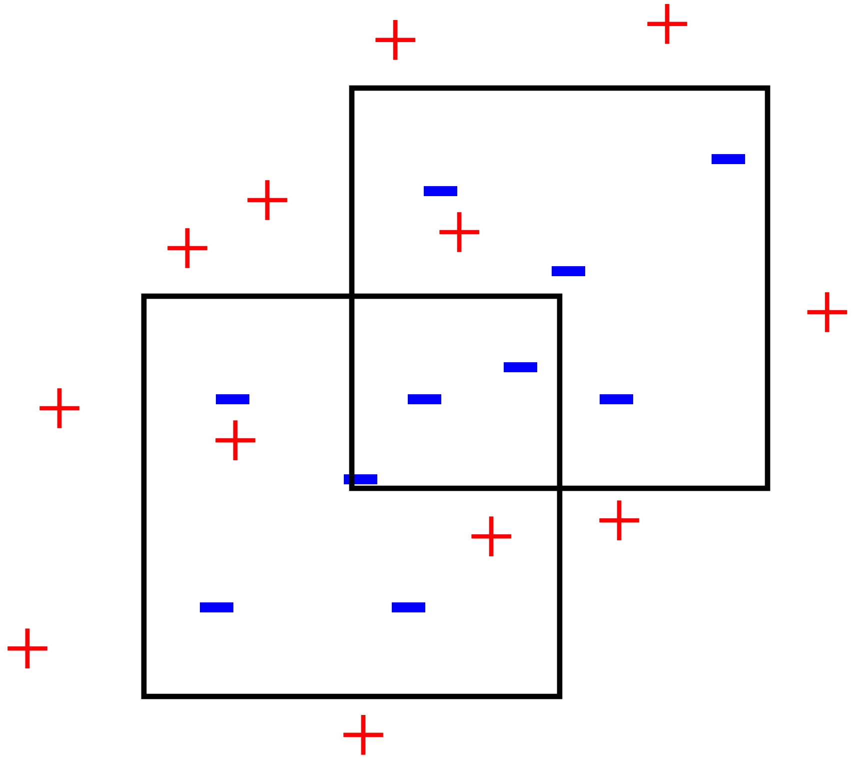

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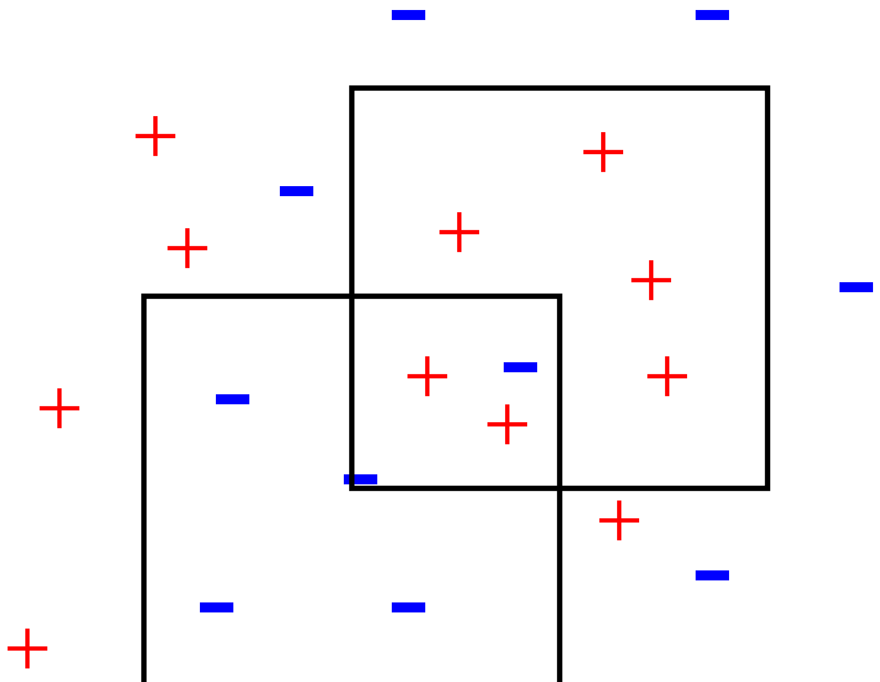

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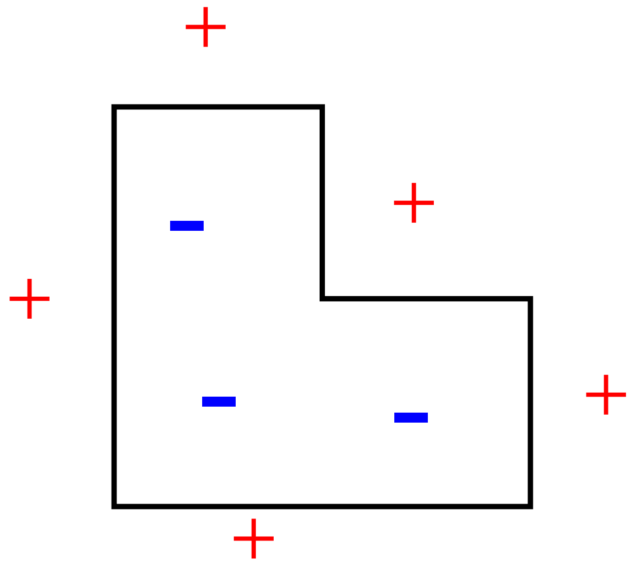
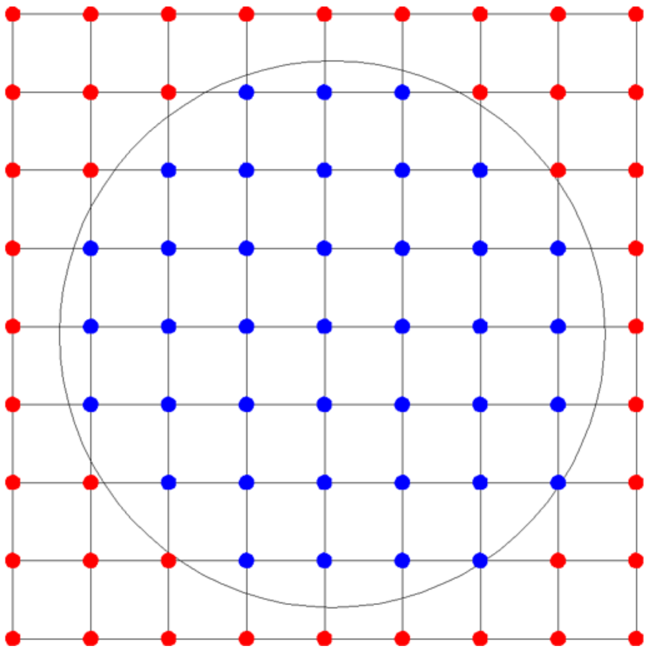


Disadvantages

- Hard to render - no polygons
- Creating polygons amounts to root finding
- Arbitrary shapes hard to represent as a function

Non-Analytic Implicit Functions

- Sample functions over grids



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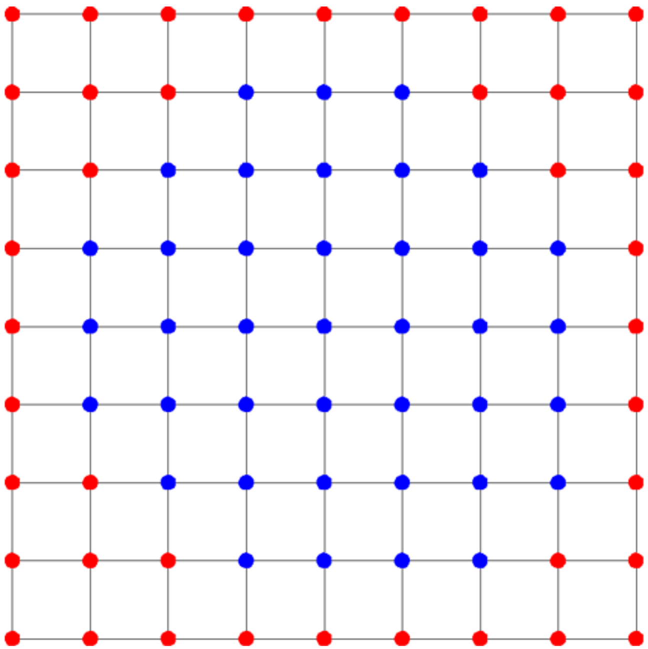
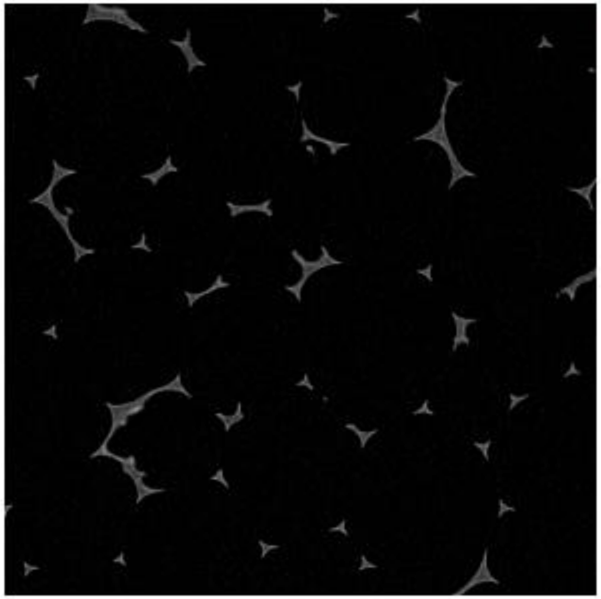


Data Sources



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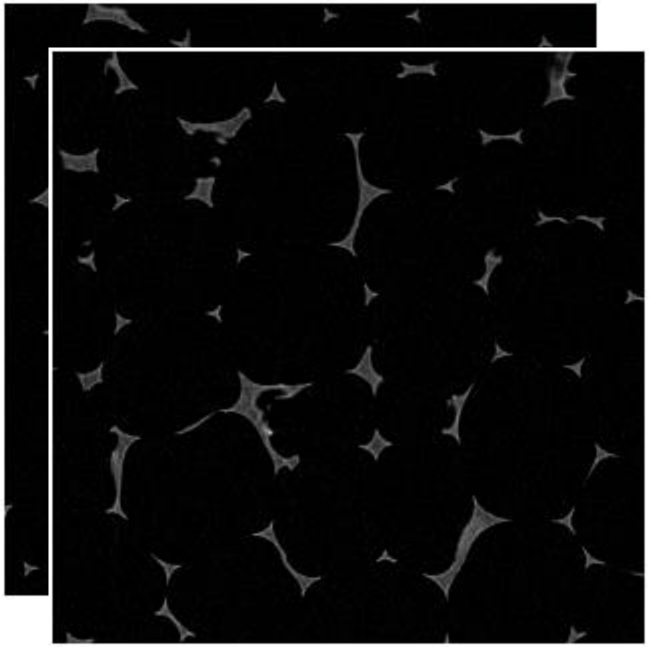

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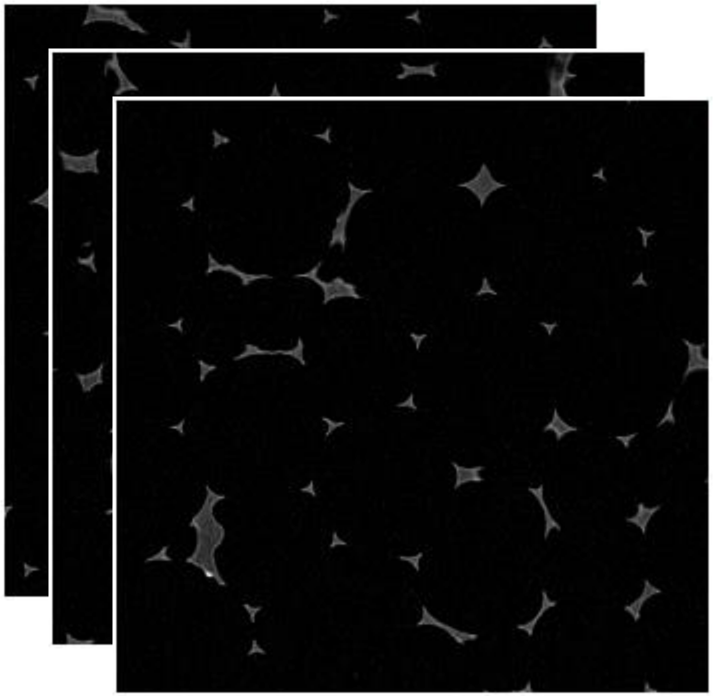

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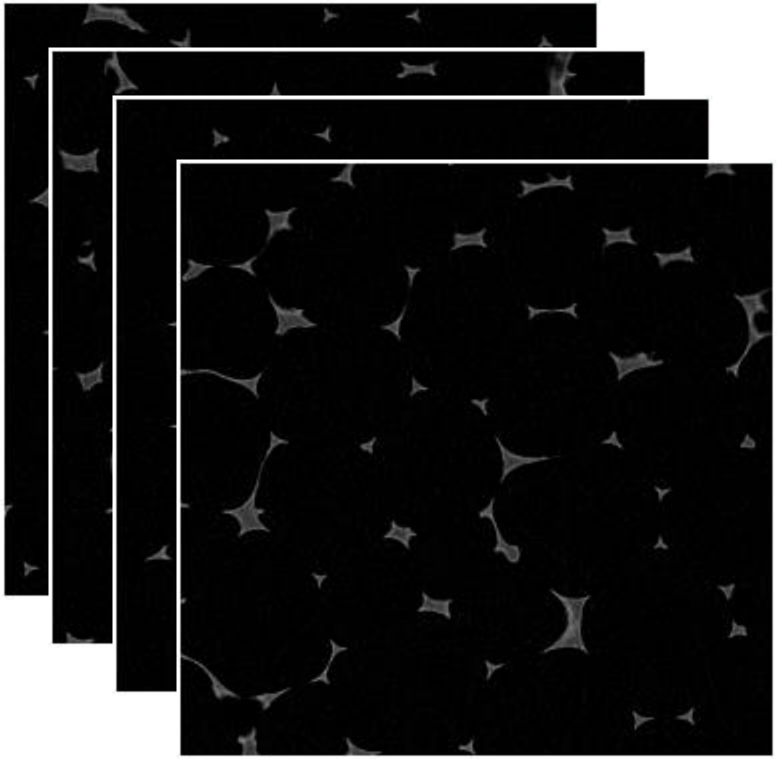

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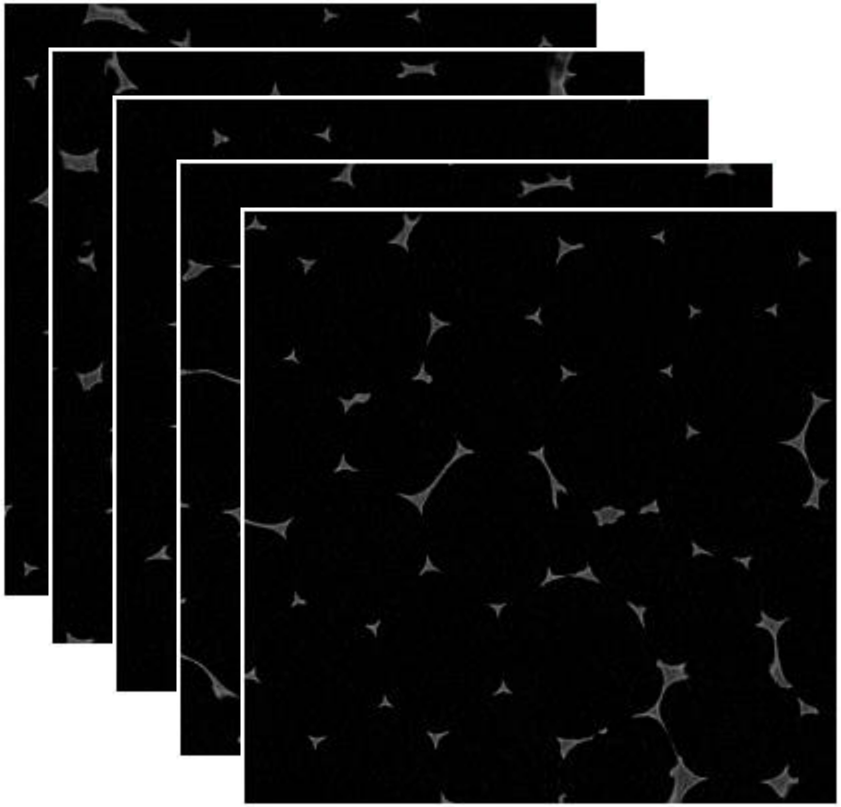

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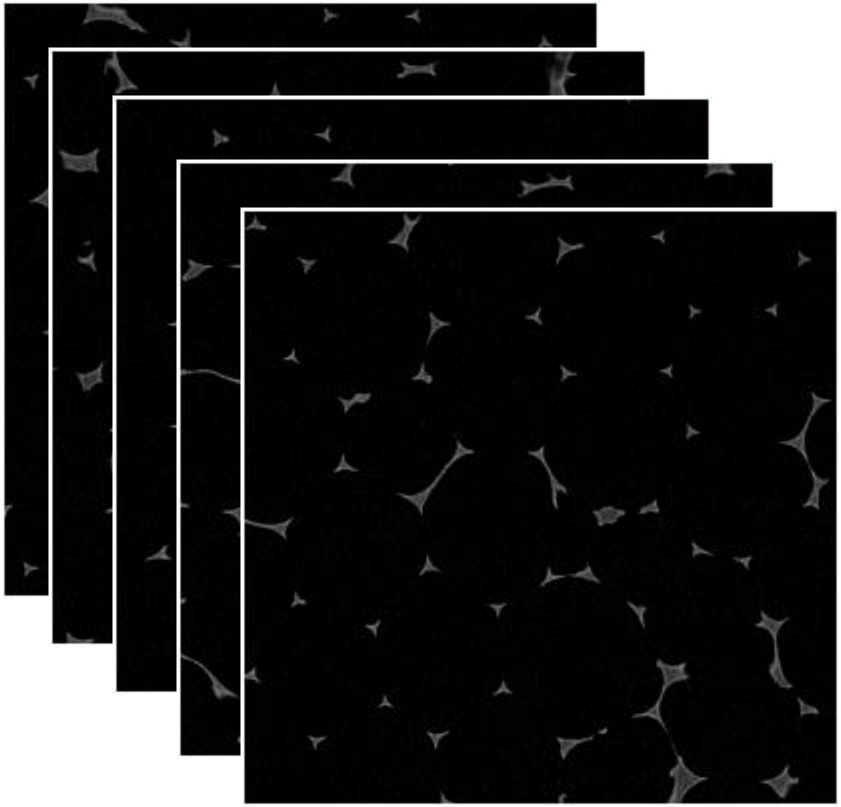
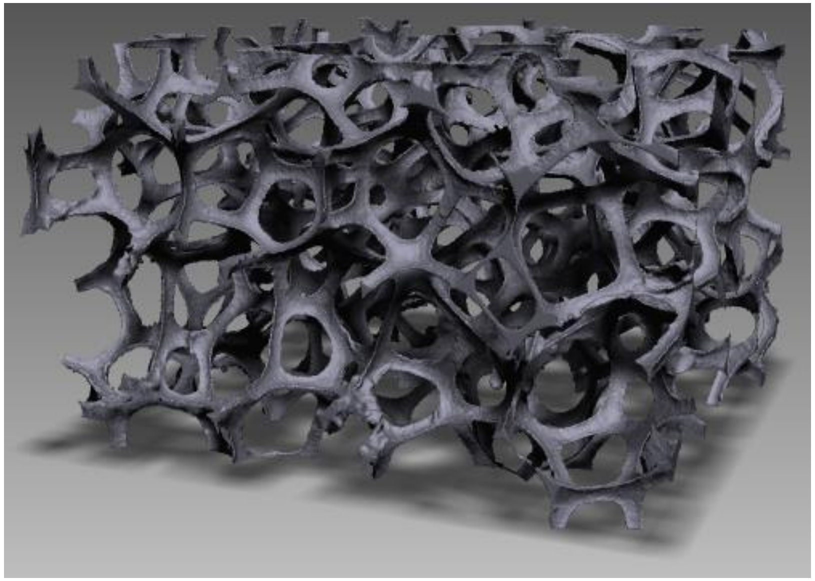
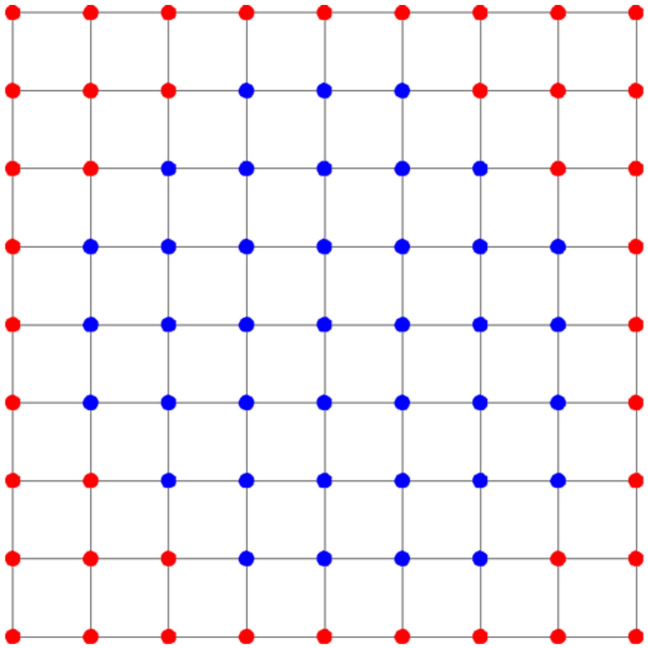




2D Polygon Generation



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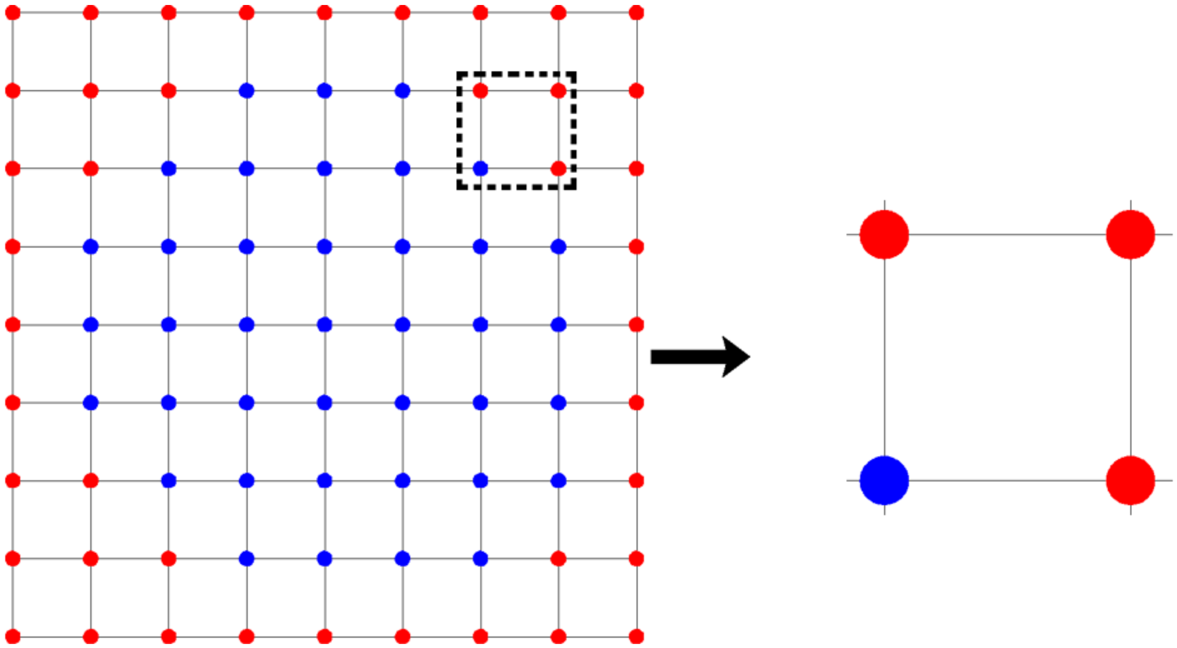

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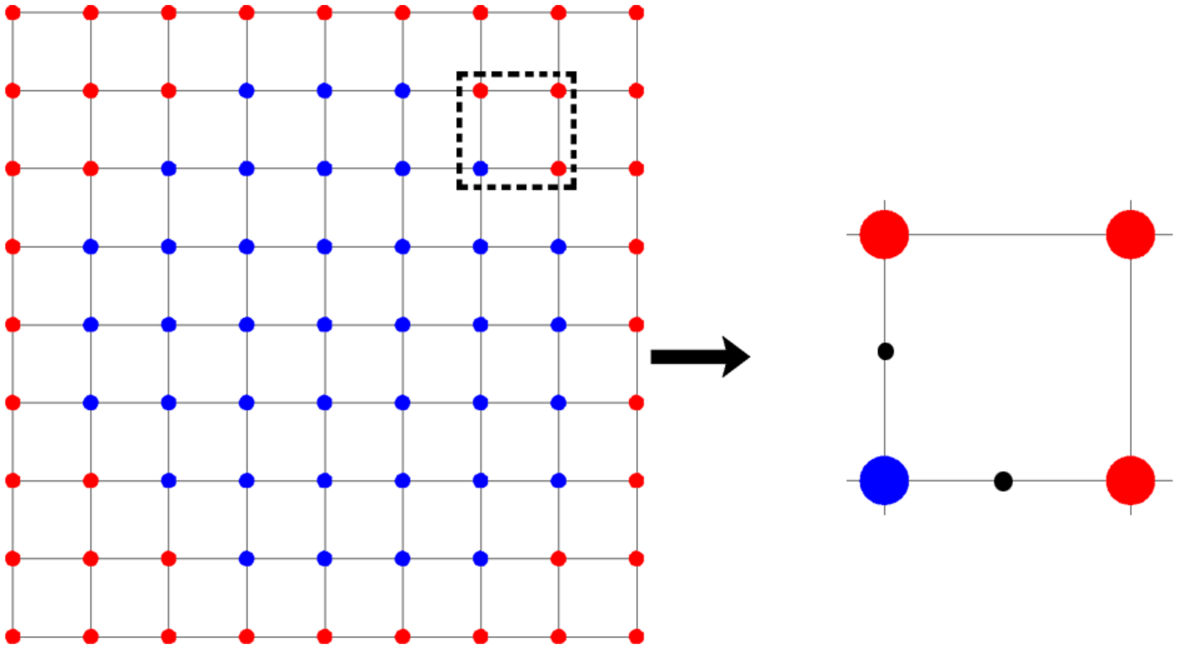

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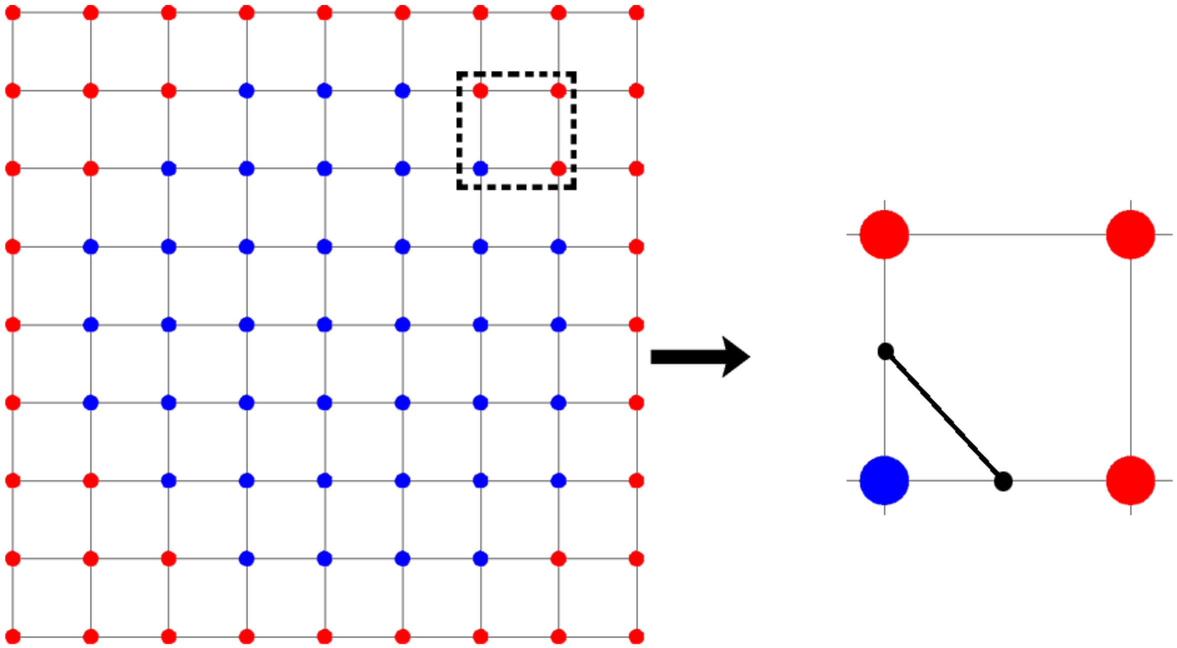

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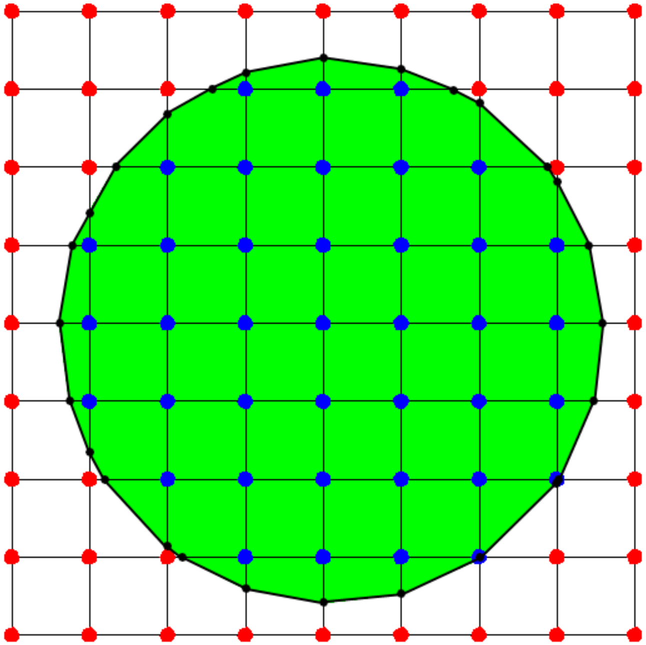

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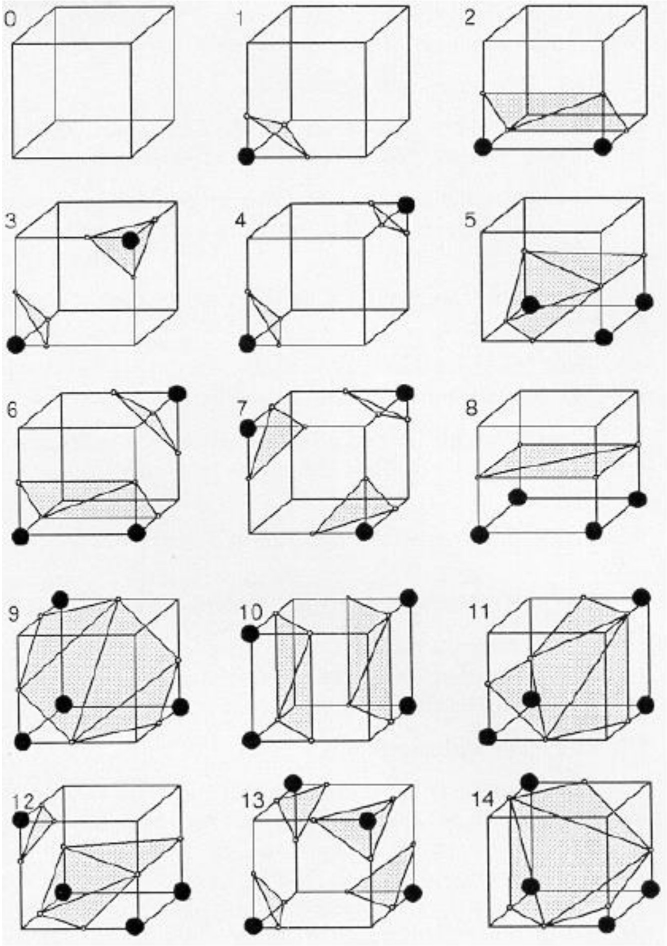

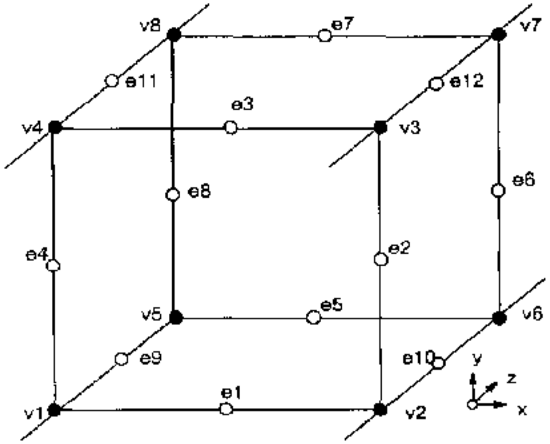

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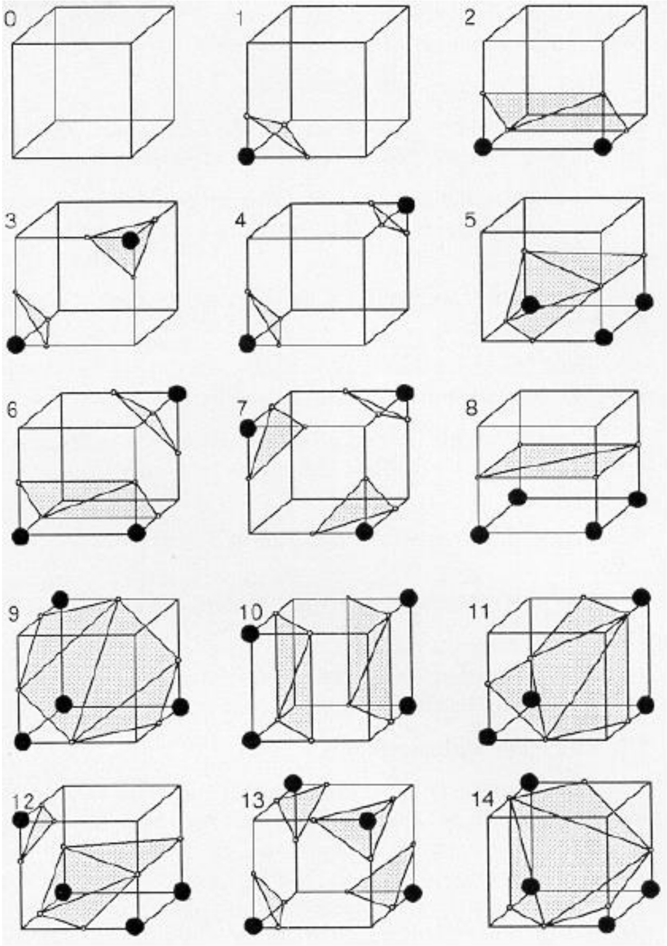

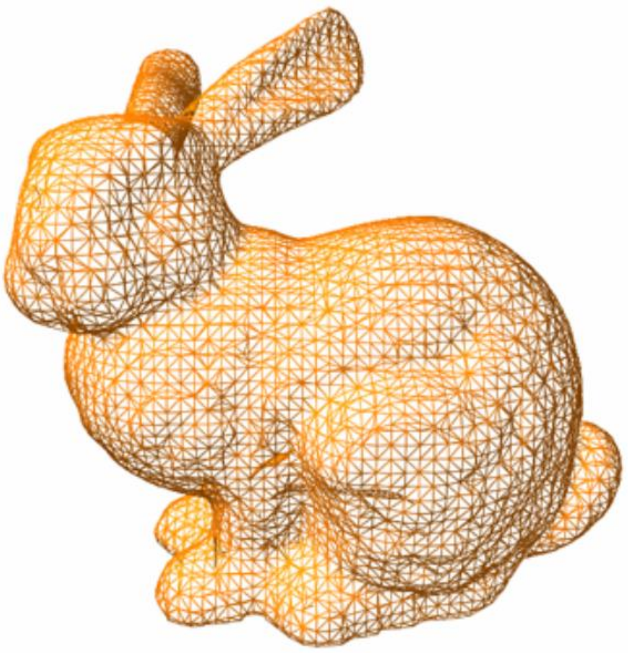

Fun Examples

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