Surfaces

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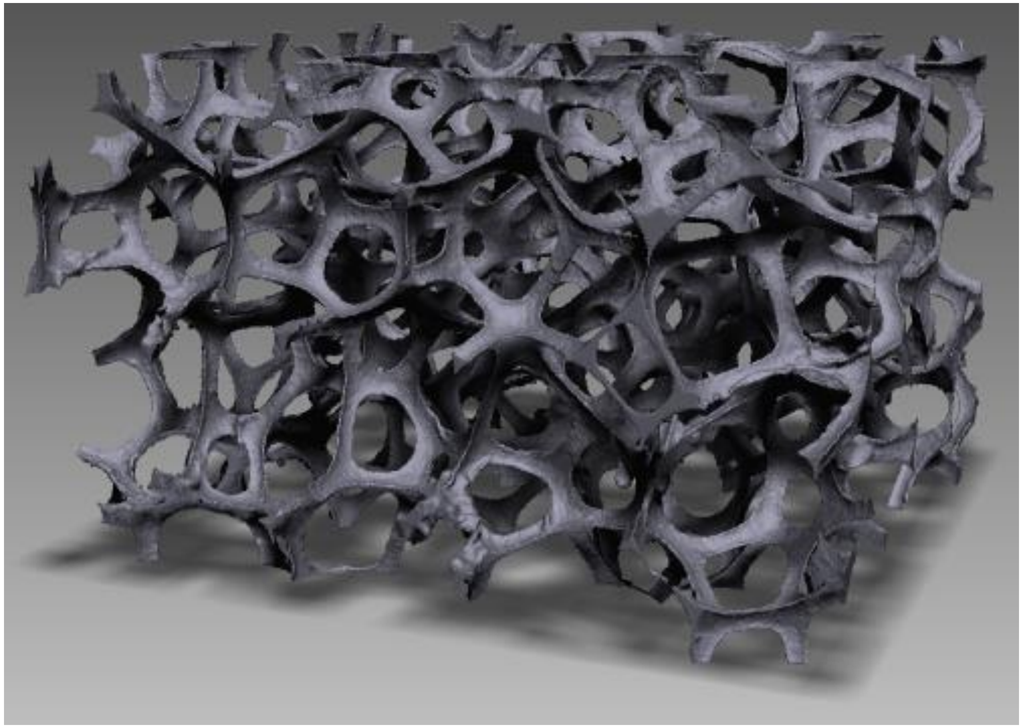
TAMU
Computer
Science

Types of Surfaces

- Implicit Surfaces
- Parametric Surfaces
- Deformed Surfaces

Implicit Surfaces

$$F(x,y,z)=0$$



Implicit Surfaces

$$x^2 + y^2 + z^2 - r^2 = 0$$

Implicit Surfaces

$$F(x, y, z) = 0$$

- Examples
  - Spheres
  - Planes
  - Cylinders
  - Cones
  - Tori

Intersecting Implicit Surfaces

$$L(t) = P + Vt$$

$$F(x, y, z) = 0$$

Intersecting Implicit Surfaces

$$L(t) = P + Vt$$

$$F(L(t)) = 0$$

Intersecting Implicit Surfaces

- Example
  - $L(t) = (0,0,-2) + (0,0,1)t$
  - $F(x, y, z) = x^2 + y^2 + z^2 - 1 = 0$

$$(0+0t)^2 + (0+0t)^2 + (-2+1t)^2 - 1 = 0$$

Intersecting Implicit Surfaces

- Example
  - $L(t) = (0,0,-2) + (0,0,1)t$
  - $F(x,y,z) = x^2 + y^2 + z^2 - 1 = 0$

$$3 - 4t + t^2 = 0$$

Intersecting Implicit Surfaces

- Example
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  - $F(x, y, z) = x^2 + y^2 + z^2 - 1 = 0$

$$t = 1, 3$$

Intersecting Implicit Surfaces

- Example
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  - $F(x, y, z) = x^2 + y^2 + z^2 - 1 = 0$

$$L(1) = (0,0,-1)$$

$$L(3) = (0,0,1)$$

Normals of Implicit Surfaces

- Given $F(x,y,z)=0$, find the normal at a point (x,y,z)

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$$\frac{\partial}{\partial t} F(x(t), y(t), z(t)) = 0$$

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- $F(x(t),y(t),z(t))=0$

$$\frac{\partial F}{\partial x}\frac{\partial x}{\partial t}+\frac{\partial F}{\partial y}\frac{\partial y}{\partial t}+\frac{\partial F}{\partial z}\frac{\partial z}{\partial t}=0$$

Normals of Implicit Surfaces

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$$\left(\frac{\partial F}{\partial x},\frac{\partial F}{\partial y},\frac{\partial F}{\partial z}\right)\cdot\left(\frac{\partial x}{\partial t},\frac{\partial y}{\partial t},\frac{\partial z}{\partial t}\right)=0$$

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Tangent of curve!!!

Normals of Implicit Surfaces

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$$\left(\frac{\partial F}{\partial x},\frac{\partial F}{\partial y},\frac{\partial F}{\partial z}\right)\cdot\left(\frac{\partial x}{\partial t},\frac{\partial y}{\partial t},\frac{\partial z}{\partial t}\right)=0$$

Normal of surface!!!

Normals of Implicit Surfaces

- Example

$$F(x,y,z)=x^2+y^2+z^2-1=0$$

Normals of Implicit Surfaces

- Example

$$F(x,y,z)=x^2+y^2+z^2-1=0$$

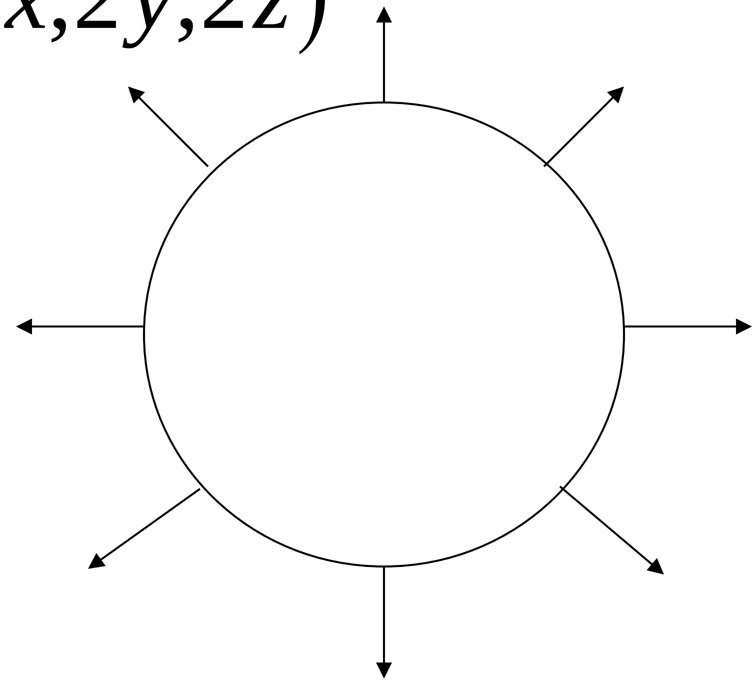
$$\left(\frac{\partial F}{\partial x},\frac{\partial F}{\partial y},\frac{\partial F}{\partial z}\right)=(2x,2y,2z)$$

Normals of Implicit Surfaces

- Example

$$F(x,y,z) = x^2 + y^2 + z^2 - 1 = 0$$

$$\left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}\right) = (2x, 2y, 2z)$$



Implicit Surfaces

- Advantages
  - Easy to determine inside/outside
  - Easy to determine if a point is on the surface
- Disadvantages
  - Hard to generate points on the surface

Parametric Surfaces

$$P(s,t) = (x(s,t), y(s,t), z(s,t))$$

Geri's Game Copyright Pixar

Parametric Surfaces

$$x(s,t) = s$$

$$y(s,t) = t$$

$$z(s,t) = s + t$$

Parametric Surfaces

$$x(s,t)=\frac{2s}{1+s^2+t^2}$$

$$y(s,t)=\frac{2t}{1+s^2+t^2}$$

$$z(s,t)=\frac{1-s^2-t^2}{1+s^2+t^2}$$

Intersecting Parametric Surfaces

$$L(t) = P(u,v)$$

- Solve three equations (one for each of x, y, z) for the parameters t, u, v
- Plug parameters back into equation to find actual intersection

Intersecting Parametric Surfaces

$$P(u,v) = (u, v, u+v)$$

$$L(t) = (0,0,-1) + (-1,0,0)t$$

Intersecting Parametric Surfaces

$$P(u, v) = (u, v, u + v)$$

$$L(t) = (0,0,-1) + (-1,0,0)t$$

$$u = -t$$

$$v = 0$$

$$u + v = -1$$

Intersecting Parametric Surfaces

$$P(u,v) = (u,v,u+v)$$

$$L(t) = (0,0,-1) + (-1,0,0)t$$

$$u = -1$$

$$v = 0$$

$$t = 1$$

Intersecting Parametric Surfaces

$$P(u,v) = (u, v, u+v)$$

$$L(t) = (0,0,-1) + (-1,0,0)t$$

$$L(1) = (-1,0,-1)$$

Normals of Parametric Surfaces

- Assume t is fixed

$$P(s,t) = F(s)$$

Normals of Parametric Surfaces

- Assume t is fixed

$$P(s,t) = F(s)$$

Curve on surface

Normals of Parametric Surfaces

- Assume *t* is fixed

$$P(s,t) = F(s)$$

$$\frac{\partial P(s,t)}{\partial s} = \frac{\partial F(s)}{\partial s}$$

Tangent at *s*

Normals of Parametric Surfaces

- Assume *s* is fixed

$$P(s,t) = G(t)$$

Normals of Parametric Surfaces

- Assume s is fixed

$$P(s,t) = G(t)$$

Curve on surface

Normals of Parametric Surfaces

- Assume s is fixed

$$P(s,t) = G(t)$$

$$\frac{\partial P(s,t)}{\partial t} = \frac{\partial G(t)}{\partial t}$$

Tangent at t

Normals of Parametric Surfaces

- Normal at s, t is

$$norm = \frac{\partial P(s,t)}{\partial s} \times \frac{\partial P(s,t)}{\partial t}$$

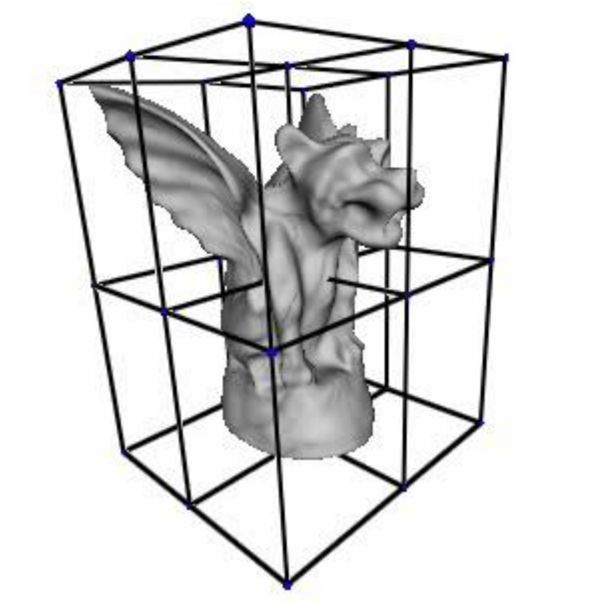
Parametric Surfaces

- Advantages
  - Easy to generate points on the surface
- Disadvantages
  - Hard to determine inside/outside
  - Hard to determine if a point is on the surface

Deformed Surfaces

- Assume we have some surface S and a deformation function $D(x,y,z)$
- $D(S)$ is deformed surface

- Useful for creating complicated shapes from simple objects

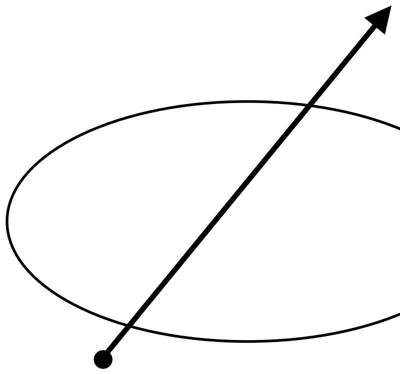


Intersecting Deformed Surfaces

- Assume $D(x,y,z)$ is simple… a matrix
- First deform line $L(t)$ by inverse of D
- Calculate intersection with undeformed surface S
- Transform intersection point and normal by D

Intersecting Deformed Surfaces

- Example

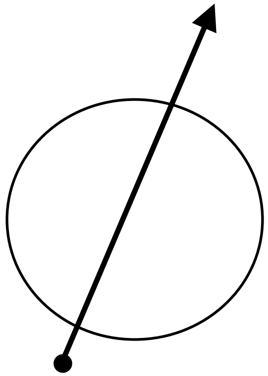


$$D(x, y) = (2x, y)$$

$$L(t) = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} t$$

Intersecting Deformed Surfaces

- Example

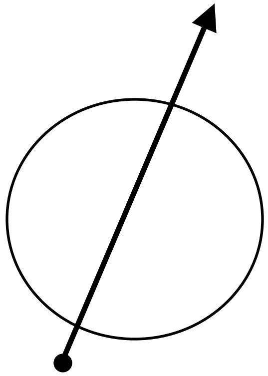


$$D(x, y) = (2x, y)$$

$$D^{-1}(L(t)) = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \left(\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} t \right)$$

Intersecting Deformed Surfaces

- Example

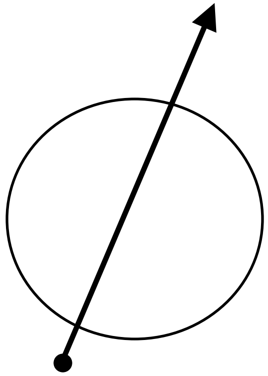


$$D(x, y) = (2x, y)$$

$$D^{-1}(L(t)) = \begin{pmatrix} -1/2 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1/2 \\ 1 \\ 0 \end{pmatrix} t$$

Intersecting Deformed Surfaces

- Example



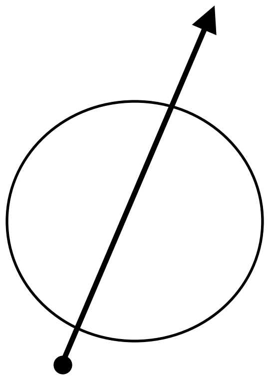
$$D(x,y) = (2x, y)$$

$$D^{-1}(L(t)) = \begin{pmatrix} -1/2 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1/2 \\ 1 \\ 0 \end{pmatrix} t$$

$$(1/2\,t - 1/2)^2 + (t-1)^2 - 1 = 0$$

Intersecting Deformed Surfaces

- Example



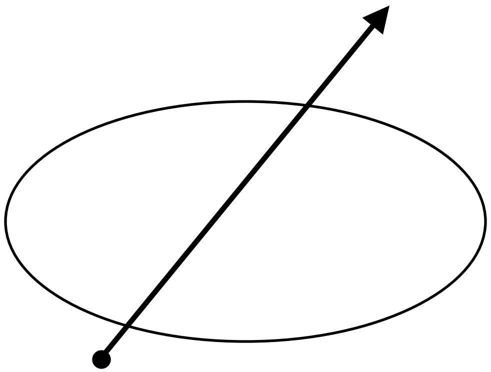
$$D(x, y) = (2x, y)$$

$$D^{-1}(L(t)) = \begin{pmatrix} -1/2 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1/2 \\ 1 \\ 0 \end{pmatrix} t$$

$$t \approx 0.11, 1.89$$

Intersecting Deformed Surfaces

- Example



$$D(x, y) = (2x, y)$$

$$L(0.11) \approx (-0.89, -0.89)$$

$$L(1.89) \approx (0.89, 0.89)$$

Normals of Deformed Surfaces

- Define how tangents transform first
- Assume curve $C(t)$ on surface

$$C'(t) \approx \frac{C(t+h) - C(t)}{h}$$

Normals of Deformed Surfaces

- Define how tangents transform first
- Assume curve $C(t)$ on surface

$$D(C)'(t) \approx \frac{D(C)(t+h) - D(C)(t)}{h}$$

Normals of Deformed Surfaces

- Define how tangents transform first
- Assume curve $C(t)$ on surface

$$D(C)'(t) = D(C(t))$$

Tangents transform by just applying the deformation *D*!!!

Normals of Deformed Surfaces

- Normals and tangents are orthogonal before and after deformation

$$N^T T = 0$$

Normals of Deformed Surfaces

- Normals and tangents are orthogonal before and after deformation

$$N^T T = 0$$

$$(MN)^T DT = 0$$

Normals of Deformed Surfaces

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$$N^T M^T D T = 0$$

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$$M^T D = I$$

Normals of Deformed Surfaces

- Normals and tangents are orthogonal before and after deformation

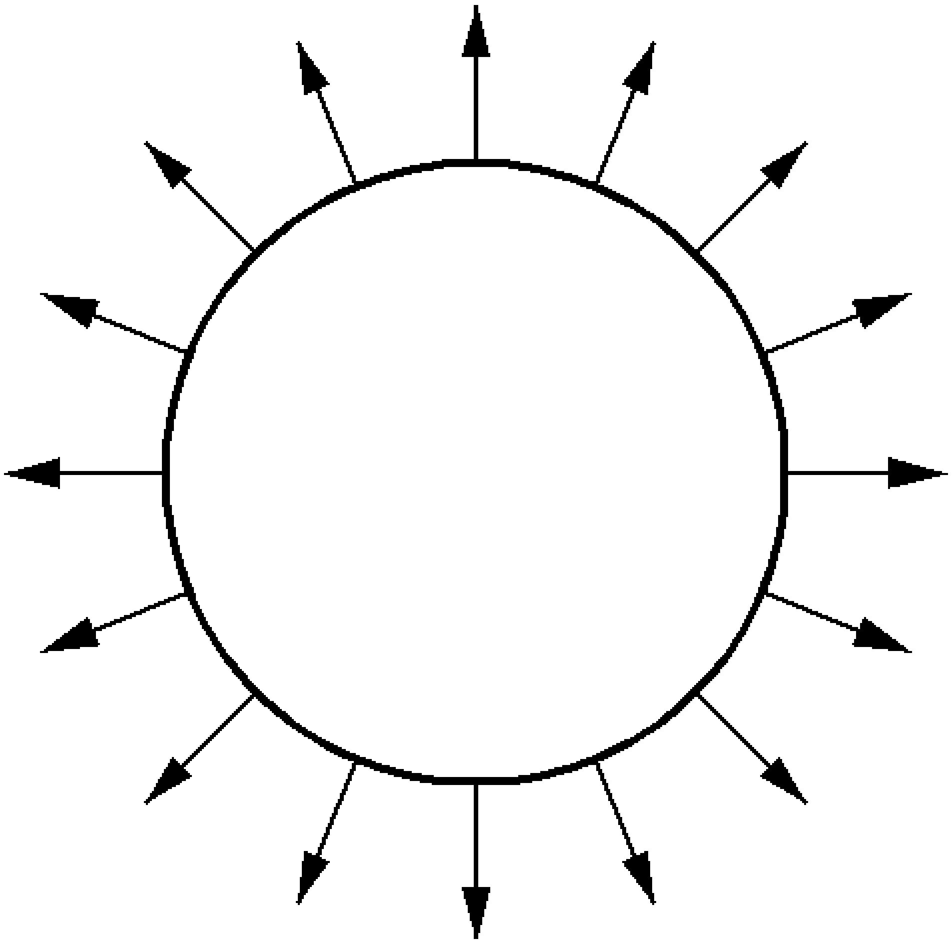
$$N^T T = 0$$

$$N^T M^T D T = 0$$

$$M = D^{-T}$$

Normals of Deformed Surfaces

- Example



Normals of Deformed Surfaces

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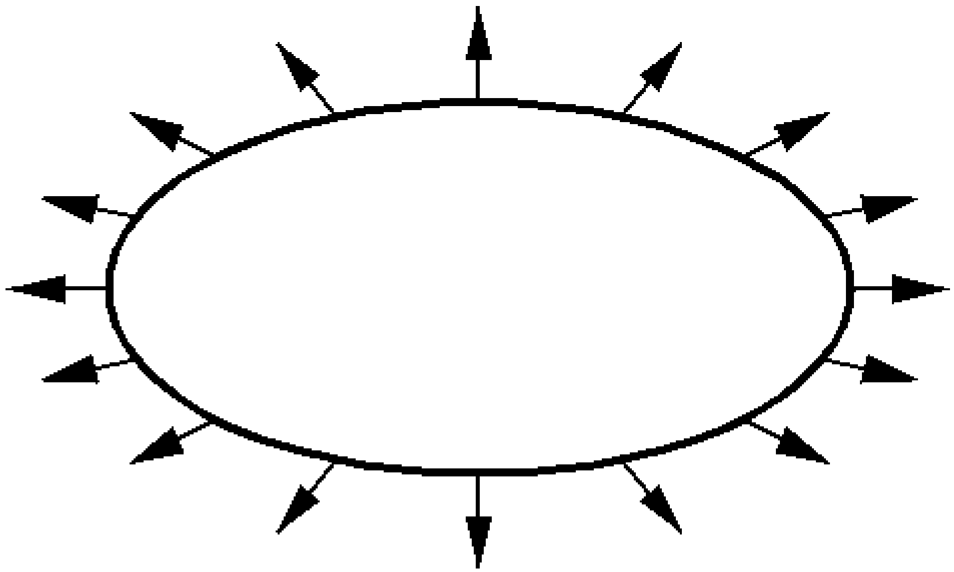

Normals of Deformed Surfaces

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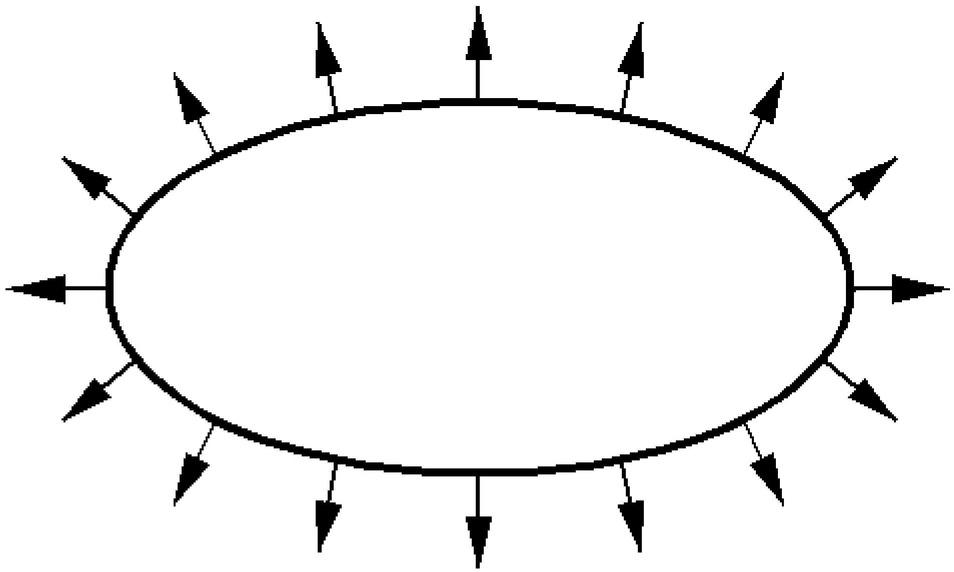

Deformed Surfaces

- Advantages
  - Simple surfaces can represent complex shapes
  - Affine transformations yield simple calculations
- Disadvantages
  - Complicated deformation functions can be difficult to use (inverse may not exist!!!)