CSCE 222 Homework 3 (Due Mar. 4)

1 and 2. Prove, for each pair of expressions $(f(n), g(n))$ below, whether $f(n)$ is O, o, Ω, ω or Θ of $g(n)$. In each case, it is possible that more than one of these conditions is satisfied.

1. $f(n) = (n + \sqrt{n}) \log n$, $g(n) = n \log(n + \sqrt{n})$.

2. $f(n) = 2^{n + \frac{1}{n}}$, $g(n) = 2^{\frac{n}{2} + \frac{2}{n}}$.

3. Compute the worst case time complexity of the following algorithm.

for $i = 1$ to 2^n do
    for $j = i$ to 2^n do
        print (i, j).

4. Prove by induction on n that $\displaystyle\sum_{k=1}^{n} k^2(k+1) = \frac{1}{12}n(n+1)(n+2)(3n+1)$.

5. Prove by induction on n that $\displaystyle\sum_{k=1}^{n} \frac{k}{k^2+1} \leq \frac{n}{2}$.