

CSC 222 Homework 4 (Due Mar. 20)

1. Find a formula for a_n that satisfies the recursive definition.

$$a_n = \begin{cases} 0 & \text{when } n = 0 \\ 2a_{n-1} + n2^n & \text{when } n > 0. \end{cases}$$

2. What is the number of one-to-one functions f from the set $\{1, 2, \dots, n\}$ to the set $\{1, 2, \dots, 2n\}$ so that at least one $1 \leq k \leq n$ is not mapped in the co-domain?
3. Find the least amount of different numbers to pick from positive integers that are at most 100 to guarantee two (possibly overlapping) pairs of numbers that differ by 1.
4. Prove that $\binom{2n}{n} \geq \binom{2n}{k}$ for $0 \leq k \leq 2n$.
5. Evaluate $\sum_{k=0}^n \frac{P(n, k)}{(k+1)!}$.