CSCE 222 Homework 6 (Due Apr. 22)

- 1. What is the number of one-to-one functions f from the set  $\{1, 2, ..., n\}$  to the set  $\{1, 2, ..., n\}$  so that there is at most one x with f(x) = x?
- 2. Given a set S, let R be the relation defined on the power set P(S) by A R B whenever  $A \subseteq B$  and  $A \neq B$  for  $A, B \subseteq S$ . Prove whether or not R is reflexive, symmetric, antisymmetric or transitive.
- 3. Given a finite set A with n elements, let R be the relation defined on the set of functions  $f : A \to A$  by fRg whenever the size of range  $\{f(x) \mid x \in A\}$  of f is the same as the size of range  $\{g(x) \mid x \in A\}$  of g. Prove that R is an equivalence relation and determine the number of equivalence classes.
- 4. Construct a deterministic finite-state automaton for the language  $L = \{w \in \{0, 1\}^* \mid w \text{ contains 000 but does not contain 111}\}.$
- 5. Give an informal description of a deterministic Turing machine for the language  $L = \{x \in \{0, 1\}^* \mid x \text{ is of the form } ww \text{ but not of the form } ww^R\}.$