LECTURE #2

INTERLUDE:
MODELING AND EVALUATION
OF RELIABLE SYSTEMS

READING:

1. K.S. Trivedi
   Probability & Statistics
   with Applications
   to Reliability, Queuing,
   and Computer Science
   Applications.
   Prentice-Hall, 1982

2. Performance Related
   Reliability
   Measures for
   Computing Systems
   P. D. Beaudry
   IEEE Trans. on Computers
   Vol 27, No 6, June 1978.
INTERLUDE:

MODELING RELIABILITY IN DISTRIBUTED SYSTEMS: THE BASICS

ASSUMPTION: Poisson distribution of failures

- Probability of failure during interval $\Delta t$ is approximately $\lambda(t) \Delta t$
  - $\lambda(t)$ = "Hazard" function
- Probability of two or more failures during $\Delta t$ is negligible.
- Failures are independent.
Define: \( m(t) = \int_0^t z(t) \, dt \)

- Prob. of \( k \) failures in interval \([0,t]\) is \( e^{-m(t)} \frac{[m(t)]^k}{k!} \)

- Expected Value
  \[
  E[k] = \sum_{k=0}^{\infty} k \frac{e^{-m(t)}[m(t)]^k}{k!} = m(t)
  \]

- Variance
  \[
  VAR[k] = E[k^2] - [E[k]]^2 = m(t)
  \]

- Reliability
  \[
  R(t) = \text{Prob}[0 \text{ failures in } [0,t]] = e^{-m(t)}
  \]

Substitute \( k = 0 \).
SPECIAL CASES:

CASE 1: HAZARD FUNCTION \( z(t) \) IS CONSTANT.

\[ z(t) = \lambda \]

\[ m(t) = \lambda t \] \text{ CONSTANT FAILURE RATE}

\text{"EXPONENTIAL"}

\[ \text{PR}[k \text{ FAILURES IN } (0,t)] = \frac{e^{-\lambda t}(\lambda t)^k}{k!} \]

\[ E[k] = \text{VAR}[k] = \lambda t \]

\[ R(t) = e^{-\lambda t} \]
OTHER CASE: \( z(t) = x t^x \) 

"COMPONENT WEAROUT +

\( m(t) = (A t)^x \)

"WEIBULL"

\[
\begin{align*}
\text{PR} [ K \text{ failures} ] &= \frac{e^{-(\eta t)^x}}{k!} \\
E[K] &= \text{var}[K] = (At)^x \\
R(t) &= e^{-(At)^x}
\end{align*}
\]

![Variations of R(t) graph for different \( \alpha \) values.](image)
Reliability of Non-redundant System

- System has "n" components

- All components needed to "survive"

- $R_{\text{system}}(t) = \prod_i R_i(t)$

- Exponential: $\prod_i e^{-\lambda_i t} = e^{-\sum \lambda_i t}$

=> Effect is summation of failure rates of individual components!
Simple Models

\[ \text{MTTF: mean time to failure} \]

\[ \text{MTTF} = \int_0^\infty R(t) \, dt \]

"Exponential": \[ \int_0^\infty e^{-\lambda t} \, dt \]

\[ = \frac{1}{\lambda} \]
ASSUMPTIONS

- System partitioned into "modules"
- Modules fail independently
- Once model fails, yields incorrect results (no Byzantine failures)
- Subsequent failures cannot bring system to functional state

EXAMPLES:
- Series-parallel systems
- n-out-of-N systems
- ....
SERIES - PARALLEL SYSTEMS

- SERIES SYSTEM

\[ R_{\text{series}}(+) = R_1(+) \times \ldots \times R_n(+) \]

- PARALLEL SYSTEM

\[ P[\text{at least one node}] = 1 - P[\text{none}] \]

\[ R_{\text{parallel}} = 1 - \prod_{i} (1 - R_i(+)) \]

- COMBINATIONS

\[ R_{S_1}(+) = 1 - (1 - R_a R_b) \frac{1}{1 - R_c R_d} \]

\[ R_{S_2}(+) = (1 - (1 - R_a)(1 - R_b)) \times (1 - (1 - R_c)(1 - R_d)) \]
Out of \( n \) we need only \( n \) to function correctly, e.g. TMR

\[
R_{TMR} = \frac{R_m^3 + (\binom{3}{2}) R_m^2 \left(1 - R_m\right)}{3}
\]

All 3 operate

Any two operate.
COMPARE SIMPLEX AND THR :

\[ R_{\text{SIMPLEX}}(+) = e^{-\lambda t} \]

\[ MTTF_{\text{SIMPLEX}} = \int_0^\infty e^{-\lambda t} \, dt = \frac{1}{\lambda} \]

\[ L_{\text{THR}} = (e^{-\lambda t})^3 + \binom{3}{1} (e^{-\lambda t})^2 (1 - e^{-\lambda t}) \]
\[ = 3e^{-2\lambda t} - 2e^{-3\lambda t} \]

\[ MTTF_{\text{THR}} = \frac{3}{2\lambda} - \frac{2}{3\lambda} = \frac{5}{6\lambda} \]

ILLUSTRATION: 

\[ R(t) \]

100% PERFECT VOTER