Overview

- Constructing a belief network
- Inference in belief networks
- Knowledge engineering

Friday:

- Reading day – no classes (I will hold my office hour though).
- AI/Robotics Seminar: 3–4pm Motion Planning HRBB 302.

Belief Network Construction

Given the nodes, we have to find which nodes are directly influenced by certain nodes, and from this, find out conditional independence.

- For example:
  \[ P(MaryCalls | JohnCalls, Alarm, Earthquake, Burglary) = P(MaryCalls | Alarm) \]

- So, even if Earthquake and Burglary can be found up-stream (e.g. Earthquake causing Alarm to go off, in turn causing MaryCalls), those events are conditionally independent from MaryCalls.

To construct a Belief Network, we need to find such dependency structure.

Joint Probability Distribution Under Conditional Independence

In Belief Networks, the joint probability is given as follows:

\[ P(x_1, \ldots, x_n) = \prod_{i=1}^{N} P(x_i | Parents(X_i)). \]

This is derived from the two following equations:

\[ P(x_1, \ldots, x_n) = P(x_n | x_{n-1}, \ldots, x_1)P(x_{n-1}, \ldots, x_1) \]
\[ = P(x_n | x_{n-1}, \ldots, x_1)P(x_{n-1} | x_{n-2}, \ldots, x_1) \]
\[ \cdots P(x_2 | x_1)P(x_1) \]
\[ = \prod_{i=1}^{N} P(x_i | x_{i-1}, \ldots, x_1) \] (1)

\[ P(X_i | X_{i-1}, \ldots, X_1) = P(X_i | Parents(X_i)) \] (2)

Belief Network Construction (cont’d)

The general procedure for Belief Network construction is as follows:

1. Choose the set of relevant variables \( X_i \) that describe the domain.
2. Choose an ordering of the variables.
3. While there are variables left:
   (a) Pick a variable \( X_i \) and add a node to the network for that variable.
   (b) Set \( Parents(X_i) \) to some minimal set of nodes already in the net such that the conditional independence property is satisfied.
   (c) Define the conditional probability table for node \( X_i \).
Evaluation of the Construction Algorithm

- Because newly added nodes cannot point to existing nodes, the resulting graph is always acyclic.
- Violation of axioms of probability is avoided.
- Compact, compared to the full joint probability table (locally structured, or sparse).
  - Belief network with \( n \) nodes (binary variables) and \( k \) parents per node has \( n \cdot 2^k \) entries in the conditional probability tables:
    \[ n \text{ nodes} \times 2^k \text{ per each node}. \]
  - Full joint probability table: \( 2^n \)

Importance of Node Ordering in Belief Network Construction

The resulting Belief Network can be vastly different when the order of insertion of nodes into the network is different.

- \textit{MaryCalls}, \textit{JohnCalls}, \textit{Alarm}, \textit{Burglary}, \textit{Earthquake}
- \textit{MaryCalls}, \textit{JohnCalls}, \textit{Earthquake}, \textit{Burglary}, \textit{Alarm}

Node Ordering and Joint Probability Tables

Even with different graphs resulting from different node ordering, you can represent the same joint probability distribution.

However, some represent the conditional independence relation much better than others.

For example, the graph above requires the same number of entries as a full joint probability table.

Improper Node Ordering Can Cause Problems

- When adding nodes in the order of \textit{MaryCalls}, \textit{JohnCalls}, ..., conditional independence does not hold:
  \[ P(\text{JohnCalls}|\text{MaryCalls}) \neq P(\text{JohnCalls}) \]
- Thus, \textit{MaryCalls} has to become a parent of \textit{JohnCalls}.
- This is because if Mary calls, it probably means that the alarm has gone off, so it makes it more likely that John calls.
Strategy for Better Node Ordering

- For a node $A$ to be a parent of another node $B$, $A$ must be added to the network before $B$ is added.

- Thus, a node that has a direct influence on other nodes should be added to the network first, i.e. add the root causes first.

Strategy for Better Node Ordering (cont’d)

When building the network, stick to a causal model, rather than the other way around (e.g. inferring cause given the effect).

Causal model is beneficial because of these reasons:

- conditional probability tables can be made smaller
- the conditional probabilities can be easier to come up with
- easier to reason about the domain using the network

Probabilistic Inference

- Diagnostic inferences: $P(Cause|Effect)$

- Causal inferences: $P(Effect|Cause)$

- Intercausal inferences: causes of a common effect (explaining away: cause has already been found)
  
  $P(Cause|Effect) \gg P(Cause|Effect \wedge OtherCause)$

- Mixed inferences: combining two or more of the above
  
  $P(A|CauseOf A \wedge EffectOf A)$

Answering Queries: A Brief Outline

Given a set of evidence $E$, find the conditional probability $P(X|E)$ where $X$ is the query.

- Recursively determine the causal support $E_X^+$ for $X$.
- Recursively determine the evidential support $E_X^-$ for $X$.

Note: This is only when the graph is singly connected.
Using Belief Networks and Probabilistic Inference

- Making decisions based on the derived probabilities and an agent's utility function.
- Deciding which additional variables to include in the model.
- Performing sensitivity analysis to find out which node is most important and thus should be more accurate.
- Explaining the results of reasoning.

Knowledge Engineering for Uncertain Reasoning

- Decide what to talk about (i.e. what to be included in the model). Gradually add more factors that can influence the current collection of events.
- Determine the variables to use and the range of the values.
- Encode general knowledge about the dependence between variables:
  1. qualitative: which variable depends on some other variable
  2. quantitative: probability value of the dependence (from experience, or from data gathered from a sample space)
- Encode a description of the specific problem instance: assign values to the variables.
- Pose queries to the inference procedure and get answers: what is the probability of X? how sensitive are the values in the conditional probability tables to perturbations?

Key Points

- Constructing a belief network: what is the procedure? why does node ordering matter? how to order the nodes?
- Inference in belief networks: what are the kinds of inference? what is the general method?
- Knowledge engineering: how to formulate the idea and design a system.

Next Time

- Homework (unification, resolution, and probabilistic reasoning)
- Project 2 description.
- Chapter 19, Neural Networks.