Problem Set 3
CPSC 629 Analysis of Algorithms
Andreas Klappenecker

The assignment is due on Wednesday, 5/27/05, before class.

A graph $G = (V, E)$ is called 3-colorable if and only if it is possible to label the vertices of $G$ with $t, f,$ or $d,$ such that no two vertices with the same label are connected by an edge in $E$.

Q1 Consider the following graph. It is easy to see that this graph is 3-colorable.

[Diagram of a graph with vertices A, B, C, M, O, P, N, Q, R, and edges connecting them.]

Assume that the vertices $A, B, C$ are assigned the label $t$ or $f$. When is it possible to label $O$ and $R$ with $t$ in a 3-coloring of this graph with $t, f,$ and $d$?

Q2 Suppose you are given another triangle (a clique with three vertices) such that the nodes are labeled with $t, f$ and $d$. How can you connect this triangle to the graph given in Q1 such that the labels of $A, B, C,$ and $R$ are either $t$ or $f$?

Q3 Show that 3-colorability of a graph is NP-complete by giving a polynomial reduction from 3SAT. In other words, given a boolean formula $p(x)$ in 3-CNF with $n$ variables and $m$ clauses, show how to define a graph that is 3-colorable if and only if $p(x)$ is satisfiable. Make sure to explain the following:
(a) Why is 3COLOR in NP?
(b) How many vertices are needed in your method?
(c) How the literals are encoded.
(d) Why your method works.
Hint: Use the gadgets given in Q1 and Q2.

Q4 Give the graph that is associated with $(x_1 \lor \overline{x_2} \lor x_3) \land (\overline{x_1} \lor \overline{x_3} \lor x_4)$ to illustrate your method, and explain.